

אביב 2006

מס' 8 מוסד מרכזי
4 (המשק)

מס' 373 א.ס.מ
מחלקת המדעים

II. נמצא את המהירות של הווקטור $\vec{U}(t)$ כאשר $\vec{U}(t)$ הוא וקטור.

1. $u(x, y, z) = x^2 + y^2 + x \cdot z$, $\vec{U}(t) = u(f(t), g(t), h(t))$,
 $f(t) = \sin 4t$, $g(t) = e^{-t}$, $h(t) = t^3$.

2. $v(x, y) = \frac{x}{y}$, $\vec{V}(t) = v(f(t), g(t))$, $f(t) = t e^{2t}$, $g(t) = \ln(t^2 + \ln 5t)$.

3. $w(u, v) = u^2 \cdot \ln v$, $Z(x, y) = w(f(x, y), g(x, y))$,
 $f(x, y) = \frac{y}{x}$, $g(x, y) = \frac{x}{y}$.

4. $u(x, y) = \ln(e^x + e^y)$, $\vec{U}(x) = u(x, v(x))$, $v(x) = x^3$.

5. $Z(t) = w(t, f(t), v(t))$, $v(t) = \sqrt{t}$, $f(t) = \frac{1}{t}$,
 $w(t, x, y) = \tan(3t + 2x^2 - y)$.

6. $Z(u, v) = w(f(u, v), g(u, v))$, $g(u, v) = u \cos v$,
 $f(u, v) = u \sin v$, $w(x, y) = \arctan \frac{x}{y}$.

7. $g(t) = t \sin t$, $f(t) = t \cdot \cos t$, $u(x, y) = e^{xy^2}$.
 $\vec{U}'(\frac{\pi}{2})$ כאשר $\vec{U}(t) = u(f(t), g(t))$.

4 גרסאות

II נבטרו חלקיו של פונקציה מורכבת, כדל השרטט (המשך)

8. הוכח כי אם $z(x,y) = f(x^2 - y^2)$ אזי

$$y \cdot \frac{\partial z}{\partial x} + x \cdot \frac{\partial z}{\partial y} = 0$$

9. הוכח כי אם $u(x,y,z) = f(x^2 z - yz)$ אזי

$$x \cdot \frac{\partial u}{\partial x} + 2y \cdot \frac{\partial u}{\partial y} - 2z \cdot \frac{\partial u}{\partial z} = 0$$

10. הוכח כי אם $u(x,y,z) = f(x-y, y-z, z-x)$ אזי

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

III נבטרו חלקיו של פונקציה מורכבת

1. $u = x^3 + 3xy^2 - 4x^2y^5 + 1$ 2. $u = \sqrt{x^2 + y^2}$

3. $u = xy + yz + zx$ 4. $u = x^m y^n$ 5. $u = \ln \sqrt{x^2 + y^2}$

6. $u = e^{x^2 y}$ 7. $u = 2x^3 y + x^2 z^3$ 8. $u = e^x \ln y + 3x + 2y - 5$

1 1 2 1 2 1

I 1. $\frac{\partial u}{\partial x} = 4x^3 - 8xy^3$ $\frac{\partial u}{\partial y} = 4y^3 - 12x^2y^2$

2. $\frac{\partial u}{\partial x} = \frac{1}{y^2} + 2ye^{2xy}$ $\frac{\partial u}{\partial y} = \frac{-2x}{y^3} + 2xe^{2xy}$

3. $\frac{\partial u}{\partial x} = \sin(2x+3y) + 2x \cos(2x+3y)$ $\frac{\partial u}{\partial y} = 3x \cdot \cos(2x+3y)$

4. $\frac{\partial u}{\partial x} = yx^{y-1}$ $\frac{\partial u}{\partial y} = x^y \ln x$

5. $\frac{\partial u}{\partial x} = 30xy(5x^2y - y^3 + 7)^2$ $\frac{\partial u}{\partial y} = 3(5x^2y - y^3 + 7)^2(5x^2 - 3y^2)$

2 חצי נוסף ונוסחה 130

מחזורי 8.16 7"3 037N
מסוף הנוסחה 8777N

2006 אב

4 סיבוב

אילול

I 6. $\frac{\partial u}{\partial x} = \frac{1}{x+y^2} + 5^{xy^2} \cdot y^2 \cdot \ln 5$, $\frac{\partial u}{\partial y} = \frac{2y}{x+y^2} + 5^{xy^2} \cdot 2xy \cdot \ln 5$

7. $\frac{\partial u}{\partial x} = y^2 (1+xy)^{y-1}$, $\ln f = y \ln(1+xy) \Rightarrow$

$$\frac{\partial u}{\partial y} = \left(\ln(1+xy) + \frac{xy}{1+xy} \right) \cdot (1+xy)^y$$

8. $\frac{\partial u}{\partial x} = \frac{1}{x+\ln y}$, $\frac{\partial u}{\partial y} = \frac{1}{(x+\ln y) \cdot y}$

10. $\frac{\partial u}{\partial x} = -\frac{e^{-x/y}}{y}$, $\frac{\partial u}{\partial y} = \frac{x \cdot e^{-x/y}}{y^2}$

9. $\frac{\partial u}{\partial x} = \frac{y \cdot u}{xz}$, $\frac{\partial u}{\partial y} = \frac{u \ln x}{z}$, $\frac{\partial u}{\partial z} = \frac{-y u}{z^2} \ln x$

11. $\frac{\partial u}{\partial x} = -\frac{y}{x^2+y^2}$, $\frac{\partial u}{\partial y} = \frac{x}{x^2+y^2}$

12. $\frac{\partial u}{\partial x} = \frac{|y|}{x^2+y^2}$, $\frac{\partial u}{\partial y} = \frac{-xy}{|y| (x^2+y^2)}$

13. $\frac{\partial u}{\partial x} = \frac{3(1+\log_y x)^2}{x \ln y}$, $\frac{\partial u}{\partial y} = -\frac{3(1+\log_y x)^2 \cdot \ln x}{y \ln^2 y}$

הערה

$$\log_y x = \frac{\ln x}{\ln y}$$

14. $\frac{\partial u}{\partial x}(1,2) = 0$, $\frac{\partial u}{\partial y}(1,2) = \frac{1}{4}$

2 חשבון דיפרנציאלי

מספר ת"ד: 378
מספר ת"פ: 1001

2006

4 שאלות

אחרי

III

6. $\frac{\partial^2 u}{\partial x^2} = 2y \cdot e^{x^2 y} (2x^2 y + 1)$ $\frac{\partial^2 u}{\partial y^2} = x^4 \cdot e^{x^2 y}$

$\frac{\partial^2 u}{\partial x \partial y} = 2x \cdot e^{x^2 y} (1 + x^2 y)$

7. $\frac{\partial^2 u}{\partial x^2} = 12xy + 2z^3$, $\frac{\partial^2 u}{\partial y^2} = 0$, $\frac{\partial^2 u}{\partial z^2} = 6x^2 z$

$\frac{\partial^2 u}{\partial x \partial y} = 6x^2$

$\frac{\partial^2 u}{\partial x \partial z} = 6x \cdot z^2$

$\frac{\partial^2 u}{\partial y \partial z} = 0$

8. $\frac{\partial^2 u}{\partial x^2} = e^x \ln y$ $\frac{\partial^2 u}{\partial x \partial y} = \frac{e^x}{y}$ $\frac{\partial^2 u}{\partial y^2} = -\frac{e^x}{y^2}$