Simulation Exam

Mark the correct answer in each part of the following questions.

1. A two-stage experiment is conducted. In the first stage, a total of $3 \cdot 6^2 = 108$ dice are rolled. Let X denote the count of dice showing a 6 in this stage. Subsequently, in the second stage, X dice are rolled. Consider the following additional three random variables:

   - Y – the number of dice showing a 6 at the second stage.
   - Z – the number of dice showing a 1 at the second stage.
   - S – the sum of the results at the second stage.

   (For example, if in the first stage 30 dice showed a 6, and in the second, five showed a 1, five a 2, ..., five a 6, then $X = 30, Y = Z = 5, S = 105$.)

   (a) The number of pairs (x, y), for which $P_{X,Y}(x, y) > 0$, is:

   (i) 108^2.
   (ii) $\binom{109}{2}$.
   (iii) 109^2.
   (iv) $\binom{110}{2}$.
   (v) None of the above.

   (b) $P(Y = X) =$

   (i) $(1/6)^{18}$.
   (ii) $(5/6)^{108}$.
   (iii) $(31/36)^{108}$.
   (iv) $(35/36)^{108}$.
   (v) None of the above.

(c) $Y \sim$

(i) $B(108, 1/36)$.

(ii) $B(18, 1/6)$.

(iii) $\bar{B}(18, 1/6)$.

(iv) $G(1/12)$.

(v) None of the above.

(d) $P(Z = 4) \approx$

(i) $e^{-1} \cdot \frac{(3/4)^4}{3!}$.

(ii) $e^{-1} \cdot \frac{(4/3)^3}{4!}$.

(iii) $e^{-4} \cdot \frac{4^3}{3!}$.

(iv) $e^{-3} \cdot \frac{3^4}{4!}$.

(v) None of the above.

(e) $P(S = 62 | X = 57, Z = 55) =$

(i) $\frac{4}{25}$.

(ii) $\frac{6}{25}$.

(iii) $\frac{4}{36}$.

(iv) $\frac{6}{36}$.

(v) None of the above.

(f) $E(S) =$

(i) $6 \cdot 9$.

(ii) $7 \cdot 9$.

(iii) $8 \cdot 9$.

(iv) $9 \cdot 9$.

(v) None of the above.

2. We toss $3n$ coins. Let X denote the number of heads in the first $2n$ tosses and Y the number of heads in the last $2n$ tosses.

   (a) $\rho(X, Y) =$

      (i) $\frac{1}{4}$.
      (ii) $\frac{1}{3}$.
      (iii) $\frac{1}{2}$.
      (iv) 1.
      (v) None of the above.

   (b) Chebyshev's Inequality implies that $P(5n/3 < 5X - 3Y < 7n/3) >$

      (i) $1 - \frac{3 \cdot 19}{2n}$.
      (ii) $1 - \frac{5 \cdot 19}{2n}$.
      (iii) $1 - \frac{7 \cdot 19}{2n}$.
      (iv) $1 - \frac{9 \cdot 19}{2n}$.
      (v) None of the above.

      Remark: We mean the best bound that can be reached. For example, if (i) is correct, then (ii)-(iv) are correct as well, but only (i) should be marked as the correct answer.

3. An urn contains 10 white and 20 black balls. Three balls are to be drawn randomly, as follows:

   - The first will be drawn and not returned.
   - The second will be drawn and returned.
   - The third will be drawn, whereby the experiment will be over.

   (a) The probability that the third ball will be white is

      (i) not well-defined.
      (ii) $\frac{1}{3}$.
      (iii) $\frac{9}{28}$.
      (iv) $\frac{8+9+10}{28+29+30}$.
      (v) None of the above.

(b) Suppose we are now after returning the second ball to the urn and before drawing the third. We did not notice the color of the first ball, but did notice that the second was white. The probability that the third ball will be white is

(i) not well-defined.

(ii) $\frac{1}{3}$.

(iii) $\frac{10}{29}$.

(iv) $\frac{281}{29^2}$.

(v) None of the above.

4. X, Y, Z are independent random variables, all distributed $G(1/2)$.

(a) $F_{X,Y}(2.5, 3.5) =$

(i) 0.

(ii) $\frac{21}{32}$.

(iii) $\frac{63}{64}$.

(iv) $\frac{105}{128}$.

(v) None of the above.

(b) $P(X + Y = Z) =$

(i) $\frac{1}{32}$.

(ii) $\frac{3}{32}$.

(iii) $\frac{1}{9}$.

(iv) $\frac{1}{8}$.

(v) None of the above.