Nudelman Nevanlinna-Pick interpolation versus Riesz-Dunford Nevanlinna-Pick interpolation: positive Pick matrices versus completely positive Pick matrices

For $\mathcal{U}$, $\mathcal{Y}$ coefficient Hilbert spaces, we let $\mathcal{S}(\mathcal{U}, \mathcal{Y})$ be the Schur class of holomorphic functions $S(z) = \sum_{n=0}^{\infty} S_n z^n$ on the unit disk $\mathbb{D}$ with values equal to contraction operators from $\mathcal{U}$ to $\mathcal{Y}$ (so $S_n \in \mathcal{L}(\mathcal{U}, \mathcal{Y})$. Nudelman (or Left-Tangential Operator-Argument (LTOA)) point-evaluation is defined as follows: one is given an operator Z on an auxiliary Hilbert spaces $\mathcal{K}$ with spectral radius less than 1 together with an operator $X \in \mathcal{L}(\mathcal{Y}, \mathcal{K})$ and defines the LTOA point evaluation by $(XS)^{\wedge L}(Z) = \sum_{k=0}^{\infty} Z^k X S_k \in \mathcal{L}(\mathcal{U}, \mathcal{K})$. The LTOA Nevanlinna-Pick interpolation problem then is: given N LTOA point-evaluation data points (X_i, Z_i) together with values $Y_i \in \mathcal{L}(\mathcal{U}, \mathcal{K})$ for $i = 1, \dots, N$, find a Schur-class function $S \in \mathcal{S}(\mathcal{U}, \mathcal{Y})$ with $(X_i S)^{\wedge L}(Z_i) = Y_i$ for $i = 1, \dots, N$. This Nudelman or LTOA interpolation problem is very convenient for giving a succinct aggregate formulation of more detailed higher-multiplicity left-tangential interpolation problems which arises in H^∞-control theory after using the Youla reparametrization of stabilizing compensators. The solution of the LTOA NP problem is in terms of positivity of the associated Pick matrix:

$$\mathbb{P}^{LTOA} = \left[\sum_{k=0}^{\infty} Z_i^k (X_i X_j^* - Y_i Y_j^*) Z_j^{*k} \right]_{i,j=1}^{N} \succeq 0.$$

The Riesz-Dunford, or more generally, tensorial-functional-calculus point evaluation is as follows: one is given an operator Z in $\mathcal{L}(\mathcal{K})$ with spectral radius , 1 and a Schur-class function $S \in \mathcal{S}(\mathcal{U}, \mathcal{Y})$ and defines $S^{\rm ten}(Z) = \sum_{k=0}^{\infty} S_k \otimes Z^k \in \mathcal{L}(\mathcal{U} \otimes \mathcal{K}, \mathcal{Y} \otimes \mathcal{K})$. The RD/tensorial NP interpolation problem is: given N such operator points $Z_i \in \mathcal{L}(\mathcal{K})$ and N associated values $\Lambda_i \in \mathcal{L}(\mathcal{U} \otimes \mathcal{K}, \mathcal{Y}, \otimes \mathcal{K})$, find an $S \in \mathcal{S}(\mathcal{U}, \mathcal{Y})$ so that $S^{\rm ten}(Z_i) = \Lambda_i$ for $i = 1, \dots, N$. As discovered by Muhly-Solel, the solution criterion involves a more refined notion of positivity, called complete positivity in the sense of Barreto-Bhat-Liebscher-Skeide: The map

$$\phi^{ten} \colon P \mapsto \left[\sum_{k=0}^{\infty} \left((I_{\mathcal{Y}} \otimes Z_i^k P Z_j^{k*}) - \Lambda_i (I_{\mathcal{U}} \otimes Z_i^k P Z_j^k) \Lambda_j^* \right) \right]_{i,j=1}^{N}$$

should be a completely positive map from $\mathcal{L}(\mathcal{K})$ into $\mathcal{L}((\mathcal{Y} \otimes \mathcal{K})^N)$. We discuss how the RD/tensorial NP problem can be reduced to a LTOA NP problem with additional parameter, and how this observation in turn can be used to arrive at an elementary proof of the solution criterion for the RD/tensorial NP problem from the solution criterion for a LTOA NP problem. We also indicate multivariable/noncommutative generalizations if time permits.