Some useful formulae

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Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

$$cosh(x) = \frac{e^x + e^{-x}}{2},$$
$$sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta)$$
$$cos(\alpha \pm \beta) = cos(\alpha)cos(\beta) \mp sin(\alpha)sin(\beta)$$
$$cos(\alpha) + cos(\beta) = 2cos(\tfrac{\alpha+\beta}{2})cos(\tfrac{\alpha-\beta}{2})$$
$$cos(\alpha) - cos(\beta) = -2sin(\tfrac{\alpha+\beta}{2})sin(\tfrac{\alpha-\beta}{2})$$
$$sin(\alpha) \pm sin(\beta) = 2sin(\tfrac{\alpha\pm\beta}{2})cos(\tfrac{\alpha\mp\beta}{2})$$
$$1 + tan^2(\alpha) = \frac{1}{cos^2(\alpha)}$$
$$1 + ctan^2(\alpha) = \frac{1}{sin^2(\alpha)}$$

$$sin(2\alpha) = 2sin(\alpha)cos(\alpha)$$
$$cos(2\alpha) = cos^2(\alpha) - sin^2(\alpha)$$
$$sin^2(\alpha) = \frac{1-cos(2\alpha)}{2}$$
$$cos^2(\alpha) = \frac{1+cos(2\alpha)}{2}$$
$$tan(2\alpha) = \frac{2tan(\alpha)}{1-tan^2(\alpha)}$$
$$ctan(2\alpha) = \frac{ctan^2(\alpha)-1}{2ctan(\alpha)}$$
$$tan\frac{\alpha}{2} = \frac{sin(\alpha)}{1+cos(\alpha)} = \frac{1-cos(\alpha)}{sin(\alpha)}$$

$\sum\limits_{k=1}^{n} q^k = \frac{q^{n+1}-q}{q-1}$, $\log_a b = \frac{ln(b)}{ln(a)}$, $(a+b)^n = \sum\limits_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$, $a^n - b^n = (a-b)\sum\limits_{k=0}^{n-1} a^k b^{n-1-k}$

$(arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$, $(arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$, $(arctan(x))' = \frac{1}{1+x^2}$

Taylor expansion with the remainder in the form of Lagrange:

$f(x) = \sum\limits_{k=0}^{n} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!} + \frac{f^{(n+1)}(\theta)(x-x_0)^{n+1}}{(n+1)!}$, $|\theta - x_0| \le |x - x_0|$.

Taylor series: $f(x) \sim \sum\limits_{n=0}^{\infty} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!}$

Basic Taylor expansions: $e^x = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!}$, $sin(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, $cos(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$,

$\frac{1}{1-x} = \sum\limits_{n=0}^{\infty} x^n$ for $|x| < 1$,

$ln(1+x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ for $|x| < 1$,

$(1+x)^\alpha = \sum\limits_{n=0}^{\infty} \frac{\alpha(\alpha-1)...(\alpha-n+1)x^n}{n!}$ for $|x| < 1$.

$\int e^x dx = e^x + C$, $\int \frac{dx}{x} = ln|x| + C$, $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C$, for $\alpha \neq -1$

$\int sin(x)dx = -cos(x) + C$, $\int cos(x)dx = sin(x) + C$, $\int \frac{dx}{cos^2(x)} = tan(x) + C$, $\int \frac{dx}{sin^2(x)} = -ctan(x) + C$,

$\int \frac{dx}{1+x^2} = arctan(x) + C$, $\int \frac{dx}{\sqrt{1-x^2}} = arcsin(x) + C$

The length of the parameterized curve, $C = \Big\{(x(t), y(t), z(t)),\ t \in [a,b]\Big\}$: $\int\limits_a^b \sqrt{(x')^2 + (y')^2 + (z')^2}dt$.

The volume of the solid of revolution obtained by the rotation of $y = f(x)$ around $\hat{x}$-axis: $\int\limits_{x_{min}}^{x_{max}} \pi f^2(x)dx$.

The volume of the solid of revolution obtained by the rotation of $y = f(x)$ around $\hat{y}$-axis: $2\pi \int\limits_{x_{min}}^{x_{max}} xf(x)dx$.

Area of the surface of revolution obtained by the rotation of $y = f(x)$ around $\hat{x}$-axis: $\int\limits_{x_{min}}^{x_{max}} 2\pi f(x)dx$.