Some useful formulae

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Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

$$sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta)$$
$$cos(\alpha \pm \beta) = cos(\alpha)cos(\beta) \mp sin(\alpha)sin(\beta)$$
$$cos(\alpha) + cos(\beta) = 2cos(\tfrac{\alpha+\beta}{2})cos(\tfrac{\alpha-\beta}{2})$$
$$cos(\alpha) - cos(\beta) = -2sin(\tfrac{\alpha+\beta}{2})sin(\tfrac{\alpha-\beta}{2})$$
$$sin(\alpha) \pm sin(\beta) = 2sin(\tfrac{\alpha\pm\beta}{2})cos(\tfrac{\alpha\mp\beta}{2})$$

$\sum\limits_{k=1}^{n} q^k = \frac{q^{n+1}-q}{q-1}$, $\quad \log_a b = \frac{ln(b)}{ln(a)}$, $\quad (a+b)^n = \sum\limits_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$, $\quad a^n - b^n = (a-b)\sum\limits_{k=0}^{n-1} a^k b^{n-1-k}$

$(arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$, $\quad (arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$, $\quad (arctan(x))' = \frac{1}{1+x^2}$

Taylor expansion with the remainder in the form of Lagrange:

$f(x) = \sum\limits_{k=0}^{n} \frac{f^{(k)}(x_0)(x-x_0)^k}{k!} + \frac{f^{(n+1)}(\theta)(x-x_0)^{n+1}}{(n+1)!}$, $|\theta - x_0| \le |x - x_0|$.

Basic Taylor expansions: $e^x = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!}$, $sin(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, $cos(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$,

$\frac{1}{1-x} = \sum\limits_{n=0}^{\infty} x^n$ for $|x| < 1$,

$ln(1+x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ for $|x| < 1$,

$(1+x)^\alpha = \sum\limits_{n=0}^{\infty} \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)x^n}{n!}$ for $|x| < 1$.

$\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$, $\quad \vec{v} \times \vec{u} = \det\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ u_x & u_y & u_z \end{pmatrix}$. Here $\hat{x} = \vec{i}$, $\hat{y} = \vec{j}$, $\hat{z} = \vec{k}$

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11}\det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12}\det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} + a_{13}\det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

The distance between the plane $\{\vec{\mathcal{N}} \cdot (x,y,z) = d\} \subset \mathbb{R}^3$ and the point $\vec{Q} = (x_0, y_0, z_0)$ is: $\frac{|d - \vec{\mathcal{N}} \cdot \vec{Q}|}{|\vec{\mathcal{N}}|}$.

The angle between the line $Q + t\vec{v}$ and the plane $\vec{\mathcal{N}} \cdot (x,y,z) = d$ is: $sin(\alpha) = |\frac{\vec{\mathcal{N}} \cdot \vec{v}}{|\vec{\mathcal{N}}| \cdot |\vec{v}|}|$.

The distance between the line $Q + t\vec{v}$ and the point P is: $\frac{|\overrightarrow{QP} \times \vec{v}|}{|\vec{v}|}$

The tangent plane to the surface $\{f(x,y,z) = 0\}$ at the point (x_0, y_0, z_0):

$$(x - x_0)\partial_x f|_{(x_0,y_0,z_0)} + (y - y_0)\partial_y f|_{(x_0,y_0,z_0)} + (z - z_0)\partial_z f|_{(x_0,y_0,z_0)} = 0.$$

Taylor-Maclaurin expansion with the remainder in the form of Lagrange:

$$f(x,y) = f(x_0,y_0) + (x - x_0, y - y_0) \cdot grad(f)|_{(x_0,y_0)} + (x - x_0, y - y_0) \cdot \partial^2 f|_c \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}, \quad \partial^2 f = \begin{pmatrix} \partial^2_{xx} f & \partial^2_{xy} f \\ \partial^2_{xy} f & \partial^2_{yy} f \end{pmatrix}$$

$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$, $\quad \int \frac{df}{dy}|_{y=y(x)} \frac{dy}{dx} dx = \int f(y)dy$

$\int \frac{dx}{\sqrt{1-x^2}} = arcsin(x) + C$, $\int \frac{dx}{1+x^2} = arctan(x) + C$, $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} ln|\frac{x-a}{x+a}| + C$