# Introduction to Singularity Theory, 201.1.0361, BGU Spring 2017

Lectures: Dmitry Kerner, (Sundays 10:00-12:00, Thursdays 16:00-17:00).
Office hours ([58], room 217, kernerdm@math.bgu.ac.il): https://www.math.bgu.ac.il/en/teaching/hours
The site of the course: *https : //www.math.bgu.ac.il/ ∼ kernerdm*

**The structure of the final grade**

There will be about 10-12 homeworks. There will be one midterm (date: ??).
The final mark is computed as: $10\%(midterm) + 90\%(final\ exam)$.
The final exams are: Moed A (7.07.2017) , Moed B (28.07.2017).

**Prerequisites**

Calculus 3 (201.1.0031), Algebraic Structures(201.1.7031).

**Overview** This is the introductory course to the singularities of maps and spaces.
The Singularity Theory began in 19'th century from the two questions:

- How does a curve look locally near its non-smooth point?
- How does the graph of a function look locally near a critical point?

By now this is an active area lying at the crossroad of Algebraic/Analytic/Differential Geometry, Algebraic Topology, Commutative Algebra. (The immediate applications in industry and applied mathematics usually go under the name "The Catastrophe Theory".)

This course is a very basic introduction, accessible to the advanced undergraduates. The course can serve as a good motivation/preparation for the subsequent solid courses in Commutative Algebra/Algebraic Geometry.

## Syllabus

(1) An introductory sketch and some motivating examples. Critical points of functions of one variable.
(2) Basic facts about analytic series in several variables. Local Rings and germs of functions/sets. Morse critical points. Degenerate critical points. Singular (non-smooth) points of curves.
(3) Unfoldings and morsifications. Finitely determined function germs.
(4) Simple singularities. Basic singularity invariants. Plane curve singularities. Decomposition into branches and Puiseux expansion.
(5) Time permitting we will concentrate on some of the following topics:
  (a) Blowups and resolution of plane curve singularities;
  (b) Basic topological invariants of plane curve singularities (Milnor fibration);
  (c) Versal deformation and the discriminant.

## Bibliography

(1) V.I. Arnold, S.M.Gusein-Zade, A.N.Varchenko, *Singularities of differentiable maps.* Volume 1. Classification of critical points, caustics and wave fronts. (Reprint of the 1985 edition.) Modern Birkhäuser Classics. Birkhäuser/Springer
(2) J.W.Bruce, P.J.Giblin *Curves and singularities. A geometrical introduction to singularity theory.* Second edition. Cambridge University Press, Cambridge, 1992.
(3) A.Dimca, *Topics on real and complex singularities. An introduction.* Advanced Lectures in Mathematics. Friedr. Vieweg & Sohn, Braunschweig, 1987
(4) W.Ebeling, *Functions of Several Complex Variables and Their Singularities*, Graduate Studies in Mathematics, Volume 83, American Mathematical Society, 2007.
(5) C.T.C.Wall, *Singular points of plane curves.* London Mathematical Society Student Texts, 63. Cambridge University Press, Cambridge, 2004.
(6) G.-M.Greuel, C.Lossen, E.Shustin, *Introduction to singularities and deformations.* Springer Monographs in Mathematics. Springer, Berlin, 2007.
(7) J.Milnor *Singular points of complex hypersurfaces*, Annals of Mathematics Studies, No. 61 Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo 1968