

Introduction to Complex Functions.EE, 201.1.0071, BGU Spring 2020

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Office hours <https://www.math.bgu.ac.il/en/teaching/hours>

The site of the course: on Moodle (primary), <https://www.math.bgu.ac.il/~kernerdm> (secondary)

The structure of the final grade

There will be about 12 homeworks (for selftraining).

The final mark is computed as: 100% final exam.

The final exams are: Moed A (21.07.2020) , Moed B (10.08.2020).

The textbooks:

- Ahlfors, *Complex analysis*.
- Nehari, *Conformal mapping*.
- J.Bak, D.J.Newman, *Complex Analysis*.
- D,Alpay, *A Complex Analysis problem book*.

The program of the course (the order of the topics is approximate and will be adjusted during the semester)

- (1) Complex numbers, open and closed sets in \mathbb{C} , (path) connectedness. The extended complex plane (Riemann sphere), $\bar{\mathbb{C}}$. e^z and De Moivre formula.
- (2) Functions of a complex variable, continuity and real-differentiability. Complex-differentiability and Cauchy-Riemann equations. Functions analytic in a domain. Basic properties of the complex derivative.
- (3) Power series. Review of convergence of functions, uniform convergence on compacts. Radius of convergence of a power series. Differentiation of power series. Analytic functions. Taylor series for e^z , $\sin(z)$, $\cos(z)$.
- (4) Curvilinear (complex) integrals. The Cauchy-Goursat theorem. The Cauchy integral formula. Morera's theorem.
Uniform convergence of analytic functions on a compact set. The maximum principle. Liouville's theorem. The Laplace transform as a analytic function on a half-plane.
- (5) Laurent series. Annulus of convergence, existence of Laurent series expansions.
- (6) Zeroes and singularities. Order of a zero and a pole. Classification of singularities. Principal singularities, the Casorati-Weierstrass theorem. The Riemann sphere and a meromorphic function.
The residue theorem. Computations of integrals.
- (7) The Argument Principle. The maximum principle. Rouché's theorem. The open mapping theorem. Hurewicz's theorem.
- (8) Harmonic functions. The harmonic conjugate. The average and maximum principles for harmonic functions. Finding harmonic functions satisfying boundary conditions (e.g. on a strip).
- (9) Conformal maps. Branches of log. Schwarz' lemma.
- (10) Stereographic projection.
Möbius transformations. Criteria for fixing the unit disk and the upper half plane. Statement of the Riemann mapping theorem. Examples of conformal maps.
Schwarz-Christoffel transformations. Applications to harmonic functions (electrostatic potential with boundary conditions). The punctured disk is not conformally equivalent to an annulus.