GEOMETRY OF THE TANGO BUNDLE

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0. Preliminaries. The Tango bundle, [2], is an indecomposable (n−1)-bundle on the projective n-space $\mathbb{P}^n$. This bundle has an interesting geometry. In particular, we study its stability and the configuration of its jump lines. In what follows V is a linear (n+1)-space and $\mathbb{P}^n=\mathbb{P}(V)$ is the projective space where all our bundles are defined, T is the tangent bundle to $\mathbb{P}^n$ and $\mathcal{O}(1)$ is the ample bundle on $\mathbb{P}^n$. For a coherent sheaf $\mathcal{F}$ on $\mathbb{P}^n$, $\mathcal{F}(k)$ will denote $\mathcal{F}\otimes\mathcal{O}(1)^{\otimes k}$.

1. First description of Tango's bundle E(1). Raising the Euler sequence of bundles over $\mathbb{P}^n$

$$0 \longrightarrow \mathcal{O}(-1) \longrightarrow V\otimes\mathcal{O} \longrightarrow T(-1) \longrightarrow 0$$

to the second power, we get

$$(1.1) \qquad 0 \longrightarrow T(-2) \longrightarrow \Lambda^2 V\otimes\mathcal{O} \longrightarrow (\Lambda^2 T)(-2) \longrightarrow 0$$

so that

$$(1.2) \qquad H^0((\Lambda^2 T)(-2))=\Lambda^2 V$$

Let G=G(1,n) be the Grassmanian of lines in $\mathbb{P}^n=\mathbb{P}(V)$, canonically imbedded into $\mathbb{P}(\Lambda^2)$ and let W be a linear subspace in Λ^2V meeting the affine cone over G only at 0. The elements of W then correspond , under the isomorphism (1.2), to non-vanishing sections of the bundle $(\Lambda^2T)(-2)$, hence they determine a trivial subbundle of it. When dimW=m=(n−1)(n−2)/2 the quotient $(\Lambda^2T)(-2)/W$ is known as the (twisted) indecomposable Tango bundle E(1). The bundles defined by W's with dimW'<m will be styled incomplete Tango bundles.

2. Second description. The set of all (complete) Tango bundles may be identified with the set of all projective subspaces $\mathbb{P}(W)$ of dimension m−1 in $\mathbb{P}(\Lambda^2V)$ not meeting the Grassmanian G=G(1,n), [3]. Note that we have a filtration $(\Lambda^2T)(-2)\supset\mathcal{O}^m\supset\mathcal{O}^{m-1}\supset\ldots$ that gives a sequence of (non-complete) Tango quotient bundles.

3. Third description. The points of Λ^2V can be viewed as antisymmetric matrices

$(n+1)\times(n+1)$. The Tango bundle, being a set of antisymmetric matrices, can be then considered as a generalization of the null-correlation bundle.

The affine cone over G consists of matrices of rank 2 and $G^2:=\{x|x\wedge x\wedge x=0\}=\{$matrices of rank$\leq 4\}$, $G^3:=\{x|\, x\wedge x\wedge x\wedge x=0\}=\{$matrices of rank$\leq 6\}$, etc. G^2 is a set theoretic sum of all tangent spaces $T_lG=\{[x]\in\mathbb{P}(\Lambda^2V)|\, x\wedge l=0\}$, where $l\in G$, in Λ^2V. On $\mathbb{P}^3$ and $\mathbb{P}^4$ we have $G^2=\Lambda^2V$ and therefore all Tango bundles are linearly equivalent, see [3]. In general, the bundles determined by W and W' are linearly equivalent iff $\mathbb{P}(W)\cap G^1$ and $\mathbb{P}(W')\cap G^1$ are linearly equivalent, as cycles on $\mathbb{P}^{m-1}=\mathbb{P}(W)=\mathbb{P}(W')$, by means of the same linear map, determined by a linear automorphism of $\mathbb{P}(\Lambda^2V)$.

4. Tango's bundle are generated by global sections. More concretely, the following sequence is exact

(4.1) $\quad 0\longrightarrow T(-2)\longrightarrow H^0(E(1))\otimes\mathcal{O}\xrightarrow{\ \rho\ } E(1)\longrightarrow 0$

where ρ is the evaluation morphism. Indeed, (4.1) occurs as the sequence of cokernels in the snake-lemma applied to the following diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & 0 & \longrightarrow & W\otimes\mathcal{O} & \longrightarrow & W\otimes\mathcal{O} & \longrightarrow & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & T(-2) & \longrightarrow & H^0(\Lambda^2T(-2))\otimes\mathcal{O} & \longrightarrow & \Lambda^2T(-2) & \longrightarrow & 0 \end{array}$$

As a corollary , we have

(4.2) $\quad H^0(E(1))=\Lambda^2V/W,\ \dim H^0(E(1))=2n-1.$

5. Zeros of sections of E(1). From (1.2) we infer readily that a section $y\wedge z\in\Lambda^2V=H^0(\Lambda^2T(-2))$ vanishes precisely on these x's that annihilate $y\wedge z$, i.e. $y\wedge z\wedge x=0$. Indeed, $y\wedge z\wedge x=0$ means that x, as a line in V, lies on the plane spanned by y and z. According to (4.2), we treat sections of E(1) as cosets in the quotient space Λ^2V/W.

(5.1). Let $s\in\Lambda^2V/W=H^0(E(1))$. Then $s(x)=0$ iff there exist $u\in\Lambda^2V=H^0(\Lambda^2T(-2))$ such that $u+W=s$ and $u(x)=0$.

Proof. The part "if" is clear. Let now $s(x)=0$. Consider the following diagram with vertical arrows being evaluation of the global sections at the point x

$$\begin{array}{ccccccccc} 0 & \longrightarrow & W & \longrightarrow & \Lambda^2V & \longrightarrow & \Lambda^2V/W & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & W\otimes\mathcal{O}(x) & \longrightarrow & \Lambda^2T(-2)(x) & \longrightarrow & E(1)(x) & \longrightarrow & 0 \end{array}$$

By considering the kernels of vertical arrows we get (5.1).

(5.2). A global section $s+W\in\Lambda^2V/W$ of the bundle E(1) vanishes on lines represented by points $l_1,\dots,l_d\in(s+W)\cap\{$ cone over G $\}$, where d is the degree of the Plucker map.

The projective version of (5.2) is

(5.3). Sections of E(1) are in natural one-to-one correspondence with projective subspaces S of $\mathbb{P}(\Lambda^2V)$ such that $\mathbb{P}(W)\subset S$, $\dim S=m+1$.

It follows that any line in $\mathbb{P}^n$ is a zero of presisely one section of E(1), and G(1,n) splits into disjoint d-element classes consisting of (not neccesary different) lines that are common zeros of one section. Hence

(5.4). For any line $L\subset\mathbb{P}^n$ we have $\dim H^0(E(1)\otimes\mathcal{J}_L)=1$, where $\mathcal{J}_L$ is the sheaf of ideals of L.

(4.2) and (5.4) then give

(5.5). The canonical restriction map $H^0(E(1))\longrightarrow H^0(E(1)_{|L})$ is an epimorphism.

Proof. Since $\Lambda^2T(-2)_{|L}$ splits into $\mathcal{O}_L(1)^{\oplus n}\oplus\mathcal{O}_L^{\oplus n(n-1)/2}$, we have, restricting the diagram defining E(1) to L, the following sequence

(5.6) $\quad 0\longrightarrow W\otimes\mathcal{O}_L\longrightarrow\mathcal{O}_L(1)^{\oplus n}\oplus\mathcal{O}_L^{\oplus n(n-1)/2}\longrightarrow E(1)_{|L}\longrightarrow 0$

So $\dim H^0(E(1)_{|L})=2n-2$ and (5.5) follows by (5.4) .

(5.7). The set $\{\, L|\, E(1)_{|L}=\mathcal{O}_L(1)\oplus\dots\oplus\mathcal{O}_L(1)\,\}$ is open in the Grassmanian G(1,n).

Proof. The bundle $\mathcal{O}(1)\oplus\dots\oplus\mathcal{O}(1)$ on $\mathbb{P}^1$ is rigid , in the sense of local deformations. The arguments used in [1] for the analytic case work in our case too. We shall show, see (6.5) below, that the set of lines as above is non-empty so that

(5.8). The general splitting type of E(1) is (1,...,1).

6. Splitting of E(1) on lines in $\mathbb{P}^n$. If $E(1)_{|L}$ is the sum of $\mathcal{O}_L(a_i)$, $i=1\dots n-1$, with $a_1\geq a_2\geq\dots\geq a_{n-1}$, we call $(a_1,\dots,a_{n-1})$ the splitting type of L.

(6.1). If $(a_1,\dots,a_{n-1})$ is the splitting type of a line L in $\mathbb{P}^n$ then $a_i\geq 0$ and $a_1+\dots+a_{n-1}=n-1$.

Proof. The sum of a_i's is n-1 because $c_1(E(1))=n-1$. By twisting the sequence (5.6) by $\mathcal{O}_L(-1)$ we get $H^1(E_{|L})=0$. On the other hand if one of a_i's was negative then we would have $\dim H^1(E_{|L}) = \dim H^1(\mathcal{O}_L(a_1-1)\oplus...\oplus\mathcal{O}_L(a_{n-1}-1)) > 0$.

(6.2). A line L is of type (1,....,1) iff for any section s of E(1), not identically zero on L, s has at most single zero on L.

Proof. If there is a summand $\mathcal{O}(a)$, $a\geq 2$ in the decompositon of $E(1)_{|L}$, then there exist a section of E(1) that vanishes along two lines meeting L, and vice versa, (5.5).

(6.3). A line is of type (1,...,1) iff $T_lG\cap\mathbb{P}(W)=\emptyset$, where l is the point on Grassmanian G corresponding to L.

Proof. Let $[w]\in\mathbb{P}(W)\cap T_lG$, w is then a sum of two primitive vectors v_1, v_2 with $v_1\wedge l= v_2\wedge l= 0$ such that $v_1+v_2\in W$. This means that the section determined by the coset v_1+W vanishes on lines v_1 and v_2 . The restriction of this section has at least a double zero on L, so that L cannot be of type (1,...,1). Conversely, if L is not of type (1,...,1), then there is a section of E(1) that vanishes on at least two lines L_1, L_2 that meet L. Then $[l_1-l_2]\in\mathbb{P}(W)\cap T_lG$.

(6.4). Remark. The lemma (6.1) holds for incomplete Tango bundles. Proof is analogous to that of (6.1).

(6.5). Observe that (6.3) gives an interesting interpretation of the jump lines: the bundle epimorphism $\wedge^2T(-2)\longrightarrow E(1)$ gives a rational map $\mathbb{P}(\wedge^2V) \longrightarrow \mathbb{P}(H^0(E(1))= \mathbb{P}^{2n-2}$, the projection along $\mathbb{P}(W)$. Since $W\cap$ {cone over G}=0, this gives a regular map $G(1,n) \longrightarrow \mathbb{P}^{2n-2}$.The jump lines (i.e. these not of type (1,...,1)) are precisely the critical points of this map. This obviously implies the existence of lines of type (1,..,1).

(6.6). If a line L is of type $(a_1,..,a_{n-1})$ and $k=\max\{ a_i \mid i=1,..,n-1 \}$, then $\dim(\mathbb{P}(W)\cap T_lG) \geq k-2$.

Proof.There exists a section of E(1) that vanishes at k distinct lines L_i, i=1,..,k, meeting L at k distinct points, (5.5). Let H be a linear subspace of V spanned by affine planes $L_1,...,L_k$. The imbedding $H\subset V$ gives $\wedge^2H\subset\wedge^2V$ and $G(1,\dim H-1)\subset G(1,n)$. The linear subspace $W'= \wedge^2H\cap W$ of $\wedge^2H$ does not intersect $G(1,\dim H-1)$ and therefore determines a non-complete Tango bundle on $\mathbb{P}(H)$ with a non-zero section having k zeros at L. Then, according to remark (6.4) we have $\dim H\geq k+2$. Let l_i , i=1,..,k, be the points in the cone over G(1,n) representing projective lines L_i , i=1,..,k. Since all L_i's meet L and span a space of dimension bigger or equal k+2, the vectors l_i, i=1,..,k must be linearly independent in $\wedge^2V$. The vectors $l_2-l_1, l_3-l_1,...,l_k-l_1$ then belong to W and span a (k-1)-linear space there. We see that $\mathbb{P}(\mathrm{span}\{ l_i-l_1 \mid i=2,..,k \})$ is contained in the common part of $\mathbb{P}(W)$ and T_lG what concludes the proof of (6.6).

A line that is of type (n-1,0,...,0) will be called an extremal one. We shall show that they correspond to maximal intersections of T_lG and $\mathbb{P}(W)$.

(6.7). $\dim(\mathbb{P}(W)\cap T_lG)\leq n-3$.

Proof. Otherwise $G\cap T_lG$ and $\mathbb{P}(W)\cap T_lG$, being respectively of dimensions n and at least n-2, meet in T_lG that contradics the choice of W.

(6.8). Being an extremal line is an closed property.

This is clear from the deformations of bundles on a projective line, see [1].

(6.9). A line L is an extremal line iff $\dim(T_lG\cap\mathbb{P}(W))= n-3$.

Proof. If L is an extremal line then (6.6) and (6.7) imply $\dim(T_lG\cap\mathbb{P}(W))=n-3$

Conversely, assume that $\dim(T_lG\cap\mathbb{P}(W)) = n-3$. We find a section of E(1) that vanishes on n-1 lines meeting L. We may assume, (6.8), that W is generic. We look for an element $s\in\wedge^2V$ such that $G\cap T_lG\cap\mathbb{P}(s+W)$ consists of n-1 points. From the maximality of the intersection $T_lG\cap\mathbb{P}(W)$ it follows that for any $[s]\in T_lG-\mathbb{P}(W)$ the set $G\cap T_lG\cap\mathbb{P}(s+W)$ is the Schubert cycle $\sigma_{n-2}(\sigma_1)^n$. The Pieri formula gives then that for a generic s it must consist of n-1 points. This concludes the proof of (6.9).

We see that on $\mathbb{P}^4$ we have a complete classification of jump lines:

(6.10). Let n=4 and L be a line on $\mathbb{P}^4$. Then:

a) L is of type (1,1,1) iff $T_lG\cap\mathbb{P}(W)=\emptyset$,

b) L is of type (2,1,0) iff $T_lG \cap \mathbb{P}(W)$ is a point,

c) L is of type (3,0,0) iff $T_lG \cap \mathbb{P}(W)$ is a line.

(6.11). The set of extremal jump lines on $\mathbb{P}^4$ is the image of $\mathbb{P}(W^*)$ under the Veronesé map $\mathbb{P}(W^*) \longrightarrow G \subset \mathbb{P}(\Lambda^2 V)$.

Proof. Let $l \in G$ be a primitive vector representing line L, and u_1, u_2 independent non-pritimitive vectors with $l \wedge u_1 = l \wedge u_2 = 0$. Then, [3], $u_1 \wedge u_1$, $u_2 \wedge u_2$, $u_1 \wedge u_2 \in \Lambda^4 V = V^*$ represent independent hyperplanes that contain L and L is their common part. In other words

$$l = ((u_1 \wedge u_1)^* \wedge (u_2 \wedge u_2)^* \wedge (u_1 \wedge u_2)^*)^*$$

where * $\Lambda^k V \longrightarrow \Lambda^{5-k} V^*$ Another pair w_1, w_2 that spans the same subspace as u_1, u_2 gives the same l. Therefore we obtain a regular map $\mathbb{P}(W^*) \longrightarrow G \subset \mathbb{P}(\Lambda^2 V)$. Since all Tango bundles on $\mathbb{P}^4$ are linearly equivalent, it sufficies to check the rest of the claim for a concrete W, see (7.2) below.

(6.12). The configuration of the jump lines on $\mathbb{P}^4$ can be obtained from that of the null-correlation bundle on $\mathbb{P}^3$. Recall that each non-zero element $w \in W$ determines a linear form $w \wedge w$, [3]. Let us call such a form admissible. Any admissible form is determined by a unique vector, up to proportionality,[3]. Let H(w) be a hyperplane in $\mathbb{P}(V)$ given by $w \wedge w$. Fixing H(w) gives a natural imbedding $G(1,3) \subset \mathbb{P}(\Lambda^2 H(w)) = \mathbb{P}^5$ in $G(1,4) \subset \mathbb{P}(\Lambda^2 V)$. Because $w \in \Lambda^2 H(w)$ and $\mathbb{P}^5 - G(1,3)$ is the moduli space for null-correlation bundles, w determines a null-correlation bundle N(w) of rank 2 on the projective 3-space H(w). For any $x \in H(w)$ let $H_{x,w}$ be its null-correlation plane through x. The jump lines of N(w) are those contained in $H_{x,w}$. For any such line L the tangent space $T_lG(1,3)$ meets w. This means that L is a jump line for E(1). Let now $H_{x,w}$, $H_{x,w'}$ be two null-correlation planes through x, corresponding to w, w', respectively. When these two planes intersect in $\mathbb{P}^4$ only at x, then each of the lines of the penciles $H_{x,w}$, $H_{x,w'}$ is a jump line of type (2,1,0). If, however, $H_{x,w}$ and $H_{x,w'}$ intersect along a line L, the tangent spaces to the two Grassmanians given by w and w' meet $\mathbb{P}(W)$ at distinct

points and then $T_lG(1,4)$ cuts $\mathbb{P}(W)$ along a projective line, so that L is of extremal type for E(1).

7. Examples.

(7.1). On $\mathbb{P}^3$ the bundle E is the null-correlation bundle and $\mathbb{P}(W)$ is a point $w \in \mathbb{P}^5 = \mathbb{P}(\Lambda^2 V)$, not belonging to the quadric G(1,3). Then a line is a jump line iff $l \wedge w = 0$ and we see that all jump lines form a hyperplane section of the Grassmanian.

(7.2). Let be a Tango bundle on $\mathbb{P}^4 = \mathbb{P}(\mathrm{span}\,(x_0, x_1, x_2, x_3, x_4))$ defined by $W = \mathrm{span}(w_1, w_2, w_3)$ with $w_1 = x_{14} - x_{23}$, $w_2 = x_{13} - x_{04}$, $w_3 = x_{03} - x_{12}$, where $x_{ij} = x_i \wedge x_j$, [3]. If w^*'s denote the dual coordinates to w's then the map defined in (6.11) can be now written in the form

$$\Sigma a_i (w_i)^* \longrightarrow (a_1)^3 x_{01} + (a_1)^2 a_2 x_{02} + ((a_1)^2 a_3 + a_1 (a_2)^2) x_{03} + a_1 a_2 a_3 x_{04} + a_1 (a_2)^2 x_{12} + ((a_2)^3 + 2 a_1 a_2 a_3) x_{13} + (a_1 (a_3)^2 + (a_2)^2 a_3) x_{14} + (a_2)^2 a_3 x_{23} + a_2 (a_3)^2 x_{24} + (a_3)^3 x_{34},$$

so it embeds $\mathbb{P}(W^*)$ as 3-Veronese manifold.

(7.3). Let, on $\mathbb{P}^5$, a Tango bundle be given by $W = \mathrm{span}(w_0, w_1, w_2, w_3, w_4, w_5)$ where $w_0 = x_{14} - x_{23}$, $w_1 = x_{13} - x_{04}$, $w_2 = x_{03} - x_{12}$, $w_3 = x_{45} - x_{02}$, $w_4 = x_{25} - x_{01}$, $w_5 = x_{35} - x_{24}$. One can see that $W \cap (\text{cone over } G) = 0$, so $\mathbb{P}(W)$ defines Tango bundle indeed. We shall show jump lines of certain types. First consider an extremal jump line. If $l_1 = x_{34}$ then $l_1 \wedge w_0 = l_1 \wedge w_1 = l_1 \wedge w_5 = 0$, so the line $L_1 = \mathbb{P}(\mathrm{span}(x_3, x_4))$ is of extremal type , (6.8).

Now let l_2 be the point x_{01} on G. The tangent space to G at this point cuts $\mathbb{P}(W)$ along the line $\mathbb{P}(\mathrm{span}(w_1, w_2))$ and the section of E(1) given by $x_3 \wedge (x_0 + x_1)$ vanishes also on lines represented by $x_0 \wedge (x_3 + x_4)$ and $x_1 \wedge (x_2 + x_3)$. These three lines meet the line L_2, so that we get the section of E(1) restricted to L_2 with three zeros. Since L_2 is not of extremal type, it is therefore of type (3,1,0,0).

Now let us consider $L_3 = \mathbb{P}(\mathrm{span}(x_1, x_2))$. The section of E(1) represented by x_{14} vanishes also on the line x_{23}, on the other hand the section represented by x_{01} vanishes also on

x_{25}, which implies that E(1) has two non-proportional sections having the same two zeros on L_3, namely they vanish at $[x_1]$ and $[x_2]$. So L_3 must be of type (2,2,0,0).

Observe that that the tangent space to G at $l_3=x_{12}$ cuts $\mathbb{P}(W)$ along the line $\mathbb{P}(\mathrm{span}(w_0,w_4))$, and the the converse to (6.6) does not hold.

The tangent space to G at $l_4=x_{03}$ meets $\mathbb{P}(W)$ at one point w_1, and we see that the section given by x_{04} vanishes also on x_{13} providing a section with two single zeros on L_4. We will see that L_4 is of type (2,1,1,0):

(7.4). If $T_lG\cap\mathbb{P}(W)$ consists of one point then L is of type (2,1,..,1,0).

Proof. Let the point [w] be the only point of $T_lG\cap\mathbb{P}(W)$ and let $\mathbb{P}(H)$ be the projective 3-space in $\mathbb{P}^n$ that corresponds to $[w\wedge w] \in G(3,n)$. Obviously $L\subset\mathbb{P}(H)$ and if there exists a section of E(1) vanishing at two different lines L_1,L_2 meeting L, then since $[l_1-l_2]=[w]$ we have $L_1,L_2\subset\mathbb{P}(H)$. We see that the sections of E(1) vanishing at least twice at L are the same as the same sections of the rank-2 Tango bundle on $\mathbb{P}(H)$ obtained from E(1) as in the proof of (6.6), so L must be of type (2,1,..1,0).

8. Stability of Tango bundles. We will use the following fact (see remark to the lemma (2.2.1) ch.2 [2]):

(8.1). If $\mathcal{F}$ is a bundle on $\mathbb{P}^n$ such that for some line L $\mathcal{F}|_L=\mathcal{O}\oplus\mathcal{O}\oplus....\oplus\mathcal{O}$ then $\mathcal{F}$ is semistable.

(8.2). Corollary. All Tango's bundle are semistable.

(8.3). If $\mathcal{F}$ is a bundle on $\mathbb{P}^n$ as in (8.1) and all the bundles $\Lambda^p\mathcal{F}$, with $1\le p\le -1+\mathrm{rank}\mathcal{F}$, have no sections, then $\mathcal{F}$ is stable.

Proof. In virtue of (8.1) it is enough to consider coherent subsheaf $\mathcal{G}$ of $\mathcal{F}$ with $c_1(\mathcal{G})=0$ and rank r, $0< r<\mathrm{rk}\mathcal{F}$. Using arguments as in the proof of lemma (1.1.7) ch.2 [ibid], we see that $\mathcal{G}\subset\mathcal{F}$ gives us imbedding $\Lambda^r\mathcal{G}\subset\Lambda^r\mathcal{F}$ and therefore gives us a section in $\Lambda^r\mathcal{F}$, contradiction.

To apply (8.3) we must compute $H^0(\Lambda^pE)$ for $1\le p\le n-2$, E being the Tango bundle.

(8.4). For any p=1,..,n-2; k=0,..p, we have $H^k(S^k(T(-3))(k-p))=0$, S^k denoting k-th symmetric power.

Proof. Raising the twisted Euler sequence

$$0\longrightarrow \mathcal{O}(-3)\longrightarrow V\otimes\mathcal{O}(-2)\longrightarrow T(-3)\longrightarrow 0$$

to the k-th symmetric power and then twisting it by $\mathcal{O}(k-p)$ we get

$$0\longrightarrow S^{k-1}(V\otimes\mathcal{O}(-2))(k-p-3)\longrightarrow S^k(V\otimes\mathcal{O}(-2))(k-p)\longrightarrow S^k(T(-3))(k-p)\longrightarrow 0$$

that is

$$0\longrightarrow \mathcal{O}(-k-p-1)^{\oplus A}\longrightarrow \mathcal{O}(-k-p)^{\oplus B}\longrightarrow S^k(T(-3))(k-p)\longrightarrow 0$$

where $A=(n+p-1)!/n!(p-1)!$, $B=(n+p)!/n!p!$. We see that all the groups $H^i(S^k(T(-3))(k-p))$ vanish for i=0,...,n-2.

(8.5). All the groups $H^0(\Lambda^pE)$ are zero for p=1,..,n-2.

Proof. Indeed, from the exact sequence (4.1) we obtain by means of standard cohomological algebra the following long exact sequence of vector bundles

$$0\longrightarrow S^p(T(-3))\longrightarrow...\longrightarrow S^k(T(-3))\otimes\Lambda^{p-k}(M\otimes\mathcal{O}(-1))\longrightarrow.....$$

$$...\longrightarrow T(-3)\otimes\Lambda^{p-1}(M\otimes\mathcal{O}(-1))\longrightarrow \Lambda^p(M\otimes\mathcal{O}(-1))\longrightarrow \Lambda^pE\longrightarrow 0$$

where $M=H^0(E(1))$. Obviously $S^k(T(-3))\otimes\Lambda^{p-k}(M\otimes\mathcal{O}(-1))=S^k(T(-3))(p-k)\otimes\Lambda^{p-k}M$, so using (8.4) and hypercohomology spectral sequence arguments we easily obtain (8.5).

Hence we have proved

(8.6). All Tango bundles are stable.

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