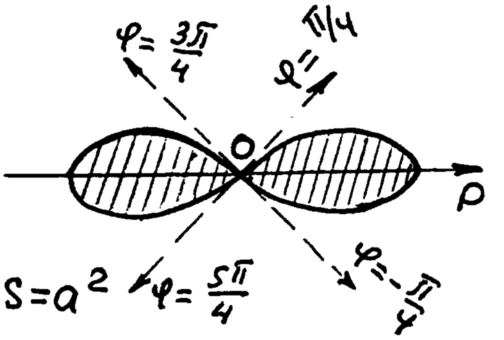
1

$r^2 = a^2\cos 2\theta \Rightarrow \cos 2\theta \ge 0 \Rightarrow$

$\Rightarrow -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}, \quad \frac{3\pi}{4} \le \theta \le \frac{5\pi}{4}$

$\frac{1}{4} S = \frac{1}{2}\int_0^{\pi/4} a^2 \cos 2\theta \, d\theta = a^2/4$

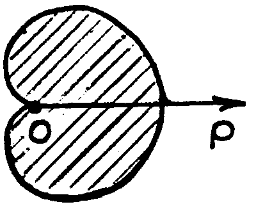
$S = a^2$



$S = a^2$

2 $\quad r = a(1+\cos\theta)$

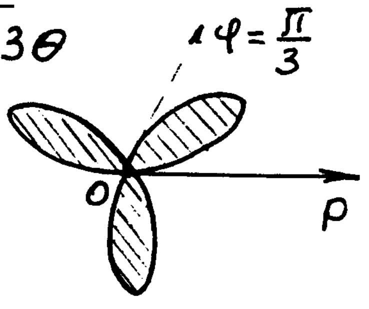
$\frac{1}{2} S = \frac{1}{2}\int_0^{\pi} a^2 (1+\cos\theta)^2 d\theta$



3 $\quad r = a\sin 3\theta \Rightarrow$

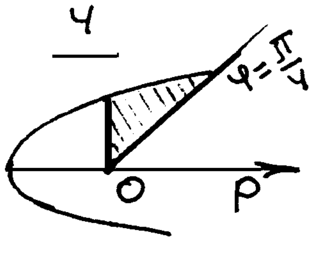
3θ

$\sin 3\theta \ge 0 \Rightarrow \left| \begin{array}{l} 0 \le \theta \le \frac{\pi}{3} \\ \frac{2\pi}{3} \le \theta \le \pi \\ \frac{4\pi}{3} \le \theta \le \frac{5\pi}{3} \end{array} \right.$



$\frac{1}{3} S = \frac{1}{2}\int_0^{\pi/3} a^2 \sin^2 3\theta \, d\theta = \frac{\pi a^2}{12} \Rightarrow S = \frac{\pi a^2}{4}$

4



$r = \frac{p}{1-\cos\theta}, \quad \theta = \frac{\pi}{4}, \quad \theta = \pi/2$

$S = \frac{1}{2}\int_{\pi/4}^{\pi/2} \frac{p^2 \, d\theta}{(1-\cos\theta)^2} = \frac{p^2}{8}\int_{\pi/4}^{\pi/2} \frac{d\theta}{\sin^4\frac{\theta}{2}}$

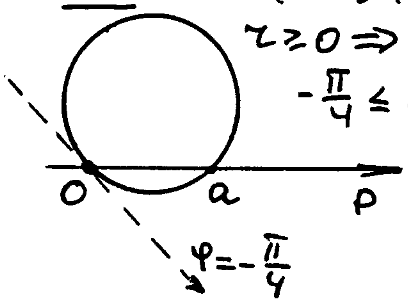
$\frac{dt}{\sin^4\frac{\theta}{2}} = \left(1+\operatorname{ctg}^2\frac{\theta}{2}\right)\frac{d\theta}{\sin^2\frac{\theta}{2}} \quad \left| \begin{array}{l} \operatorname{ctg}\frac{\theta}{2} = t \\ -\frac{d\theta}{2\sin^2\frac{\theta}{2}} = dt \end{array} \right| = -2(1+t^2)\,dt$

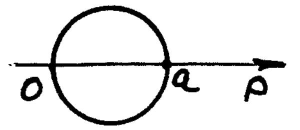
$S = -\frac{p^2}{4}\int_{\sqrt{2}+1}^{1} (1+t^2)\,dt$

5 $r = a(\cos\theta + \sin\theta)$

$r \geq 0 \Rightarrow \cos\theta + \sin\theta \geq 0$

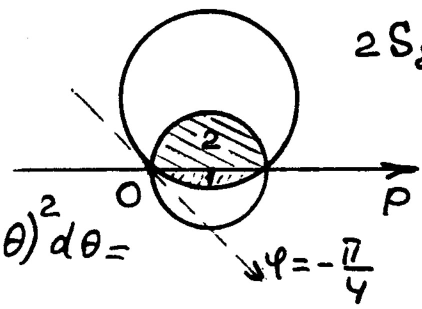
$-\frac{\pi}{4} \leq \theta \leq \frac{3}{4}\pi$





$r = a\cos\theta$

$2S_2 = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

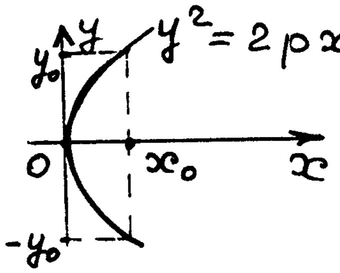


$$S_1 = \frac{1}{2}\int_{-\pi/4}^{0} a^2(\cos\theta + \sin\theta)^2\, d\theta =$$

$$= \frac{a^2}{2}\int_{-\pi/4}^{0}(1 + 2\cos\theta\sin\theta)\,d\theta = \frac{a^2}{2}\left(\theta + \sin^2\theta\right)\Big|_{-\pi/4}^{0} = \frac{a^2}{8}(\pi - 2)$$

$$S = S_1 + S_2 = \frac{a^2}{4}(\pi - 1)$$

7



$y^2 = 2px$

$$\frac{1}{2}\ell = \int_0^{y_0}\sqrt{1 + x'^2(y)}\,dy =$$

$$= \int_0^{y_0}\sqrt{1 + \frac{y^2}{p^2}}\,dy = \int_0^{y_0}\sqrt{p^2 + y^2}\,dy$$

$$\int \underbrace{\sqrt{p^2+y^2}}_{u}\,\underbrace{dy}_{dv} = \frac{y}{2}\sqrt{p^2+y^2} + \frac{p^2}{2}\ln\left|y + \sqrt{p^2+y^2}\right| + c,$$

8 $y = a\,\dfrac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{2}$, $\quad y' = \frac{1}{2}\left(e^{x/a} - e^{-x/a}\right)$

$$\ell = \int_0^{b}\sqrt{1 + \frac{1}{4}\left(e^{2x/a} - 2 + e^{-2x/a}\right)}\,dx =$$

$$= \int_0^{b}\frac{1}{2}\left(e^{x/a} + e^{-x/a}\right)dx = \frac{a}{2}\left(e^{x/a} - e^{-x/a}\right)\Big|_0^{b} =$$

$$= \frac{a}{2}\left(e^{b/a} - e^{-b/a}\right)$$

9

$$\ell = \int_0^{x_0}\underbrace{\sqrt{1+e^{2x}}}_{t}\,dx = \int_{\sqrt{2}}^{\sqrt{1+e^{2x_0}}}\frac{t^2}{t^2-1}\,dt = \left(t - \frac{1}{2}\ln\left|\frac{1+t}{1-t}\right|\right)\Bigg|_{\sqrt{2}}^{\sqrt{1+e^{2x_0}}}$$