אינטגרל בלתי מסוים
תשובות

I

1) $-\dfrac{7}{5}e^{-5x}-4x\sqrt{3x}-\dfrac{\pi x^9}{9}+\dfrac{27}{4}\sqrt[3]{x^4}-2x+C$ 2) $5\ln|x|+\dfrac{7}{3}\cot 3x+\dfrac{5}{4}\tan 4x+C$

3) $6\sin\dfrac{x}{3}+\dfrac{\cos 6x}{24}+C$ 4) $\dfrac{1}{22}(2x-3)^{11}+C$ 5) $\dfrac{\sqrt{6}}{6}\arctan\dfrac{x\sqrt{6}}{2}+C$ 6) $\dfrac{\sqrt{3}}{3}\arcsin\dfrac{x\sqrt{6}}{2}+C$

7) $\dfrac{\sqrt{3}}{3}\ln\left|\sqrt{3}x+\sqrt{3x^2-2}\right|+C$ 8) $\dfrac{\sqrt{6}}{12}\ln\left|\dfrac{x\sqrt{3}+\sqrt{2}}{x\sqrt{3}-\sqrt{2}}\right|+C$ 9) $\dfrac{1}{4}\sin 2x-\dfrac{1}{16}\sin 8x+C$

10) $3\sin\dfrac{x}{6}+\dfrac{3}{5}\sin\dfrac{5x}{6}+C$ 11) $C-\dfrac{1}{4}\cos(2x+a)-\dfrac{\sin a}{2}x$ 12) $\dfrac{1}{12}\sin 6x+\dfrac{1}{2}x+C$

13) $x-\dfrac{1}{x}-2\ln|x|+C$ 14) $\dfrac{2}{3}x\sqrt{x}+2\sqrt{x}+C$ 15) $\dfrac{4}{5}x\sqrt[4]{x}-\dfrac{24}{17}x\sqrt[12]{x^5}+\dfrac{4}{3}\sqrt[4]{x^3}+C$

16) $2x-\dfrac{12}{5}\sqrt[6]{72x^5}+\dfrac{3}{2}\sqrt[3]{9x^2}+C$ 17) $\ln|x|-\dfrac{1}{4x^4}+C$ 18) $C-x+0.5\ln\left|\dfrac{1+x}{1-x}\right|$

19) $\arcsin x+\ln(x+\sqrt{1+x^2})+C$ 20) $\dfrac{4^x}{\ln 4}+2\cdot\dfrac{6^x}{\ln 6}+\dfrac{9^x}{\ln 9}+C$

21) $-\dfrac{2}{\ln 5}0.2^x+\dfrac{1}{5\ln 2}0.5^x+C$ 22) $0.5e^{2x}-e^x+x+C$ 23) $5\cosh x-3\sinh x+C$

24) $-x-\cot x+C$ 25) $-x+\tan x+C$ 26) $-0.25(1-3x)^{4/3}+C$

27) $-0.4\sqrt{2-5x}+C$ 28) $-2.5(1-x)^{2/5}+C$ 29) $-0.2\cos 5x-x\sin 5\alpha+C$

30) $-0.5\cot(2x+\pi/4)+C$ 31) $\tan(x/2)+C$ 32) $-\cot(x/2)+C$

33) $\dfrac{(1-x)^{12}}{12}-\dfrac{(1-x)^{11}}{11}+C$ 34) $0.5x^2-x+\ln|1+x|+C$

35) $\dfrac{1}{3}\left((x+1)^{3/2}-(x-1)^{3/2}\right)+C$ 36) $-\dfrac{8+30x}{375}(2-5x)^{3/2}+C$

37) $C-x(1-\ln x)$ 38) $C-(x+1)e^{-x}$ 39) $C-\dfrac{1}{3}x^2\cos 3x+\dfrac{2}{27}\cos 3x+\dfrac{2}{9}x\sin 3x$

40) $x\arcsin 2x+\dfrac{1}{2}\sqrt{1-4x^2}+C$ 41) $C-\dfrac{3}{2}x+\dfrac{9+x^2}{2}\arctan\dfrac{x}{3}$

42) $\dfrac{3\sin 5x-5\cos 5x}{34}e^{3x}+C$ 43) $\dfrac{3\sin 3x-2\cos 3x}{13}e^{-2x}+C$

44) $\dfrac{x}{2}\sqrt{5-x^2}+\dfrac{5}{2}\arcsin\dfrac{x}{\sqrt{5}}+C$ 45) $\dfrac{x}{2}\sqrt{x^2+8}+4\ln\left|x+\sqrt{x^2+8}\right|+C$

46. a) $\dfrac{1}{22}(2x-3)^{11}+C$ b) $C-\sqrt{1-x^2}$ c) $0.25\sqrt[3]{(1+x^3)^4}+C$

d) $C-0.25\ln|3-2x^2|$ e) $C-0.5/(1+x^2)$

47. a) $0.25\arctan\dfrac{x^2}{2}+C$ b) $\dfrac{\sqrt{2}}{16}\ln\left|\dfrac{x^4-\sqrt{2}}{x^4+\sqrt{2}}\right|+C$

48. a) $C-\dfrac{1}{2}e^{-x^2}$ b) $\cos\dfrac{1}{x}+C$ c) $\dfrac{1}{8}\sqrt[3]{8x^3+27}+C$ d) $\dfrac{\ln^3 5x}{3}+C$ e) $\ln|\ln(\ln x)|+C$

$f)\,0.5\ln\left|\dfrac{1+\sin x}{1-\sin x}\right|+C$ or $\ln\left|\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right|+C$

$g)\,\dfrac{1}{2}\arctan^2 x+C$ $h)\,C-\dfrac{1}{\arcsin x}$ $i)\,C-\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$ $j)\,C+\tan\dfrac{x}{2}$

$49.\,a)\,\arcsin x-\sqrt{1-x^2}+C$ $b)\,\dfrac{3}{2}\ln(x^2+9)-\dfrac{1}{3}\arctan\dfrac{x}{3}+C$

$c)\,C-8\sqrt{5+2x-x^2}-3\arcsin\dfrac{x-1}{\sqrt{6}}$ $d)\,0.5\ln(x^2+2x+2)+\arctan(x+1)+C$

$e)\,3\sqrt{x^2+2x+2}-4\ln\left|x+1+\sqrt{x^2+2x+2}\right|+C$ $f)\,\ln\dfrac{(x-4)^2}{|x-3|}+C$

50. $a)\,\dfrac{1}{12}\sin^6 2x+C$ $b)\,C-\dfrac{1}{5}\ln|\cos 5x|$ $c)\,C+\dfrac{1}{3}\ln|\sin 3x|$ $d)\,\dfrac{2}{\sqrt{\cos x}}+C$

$e)\,\dfrac{1}{2}\ln\left|\dfrac{1-\cos x}{1+\cos x}\right|+C$ or $\ln\left|\tan\dfrac{x}{2}\right|+C$

$51.\,a)\,\dfrac{1}{4}\sin 2x-\dfrac{1}{16}\sin 8x+C$ $b)\,3\sin\dfrac{x}{6}+\dfrac{3}{5}\sin\dfrac{5x}{6}+C$ $c)\,C-\dfrac{1}{4}\cos(2x+a)-\dfrac{\sin a}{2}x$

$d)\,\dfrac{1}{12}\sin 6x+\dfrac{1}{2}x+C$ $e)\,C-\dfrac{1}{5}\cos 5x+\dfrac{1}{15}\cos^3 5x$

$f)\,\dfrac{1}{8}\sin 4x+\dfrac{1}{64}\sin 8x+\dfrac{3}{8}x+C$ $g)\,\dfrac{1}{14}\tan^2 7x+\dfrac{1}{7}\ln|\cos 7x|+C$

$52.\,a)\,\ln|(x-2)(x+5)|+C$ $b)\,C-x+\dfrac{1}{2}x^2+\dfrac{8}{3}\ln|x+2|+\dfrac{1}{3}\ln|x-1|$

$c)\,x+\dfrac{1}{6}\ln|x|-\dfrac{9}{2}\ln|x-2|+\dfrac{28}{3}\ln|x-3|+C$ $d)\,\dfrac{1}{x+1}+\dfrac{1}{2}\ln|x^2-1|+C$

$e)\,\dfrac{1}{3}\ln|x-1|-\dfrac{1}{6}\ln(x^2+x+1)+\dfrac{1}{\sqrt{3}}\arctan\dfrac{2x+1}{\sqrt{3}}+C$

$53)\,2\sqrt{x}+2\ln\left|\sqrt{x}-1\right|+C$ $54)\,4\sqrt{x-2}-16\ln\left|\sqrt{x-2}+4\right|+C$

$55)\,1.5\ln\left|\sqrt[3]{x^2}+1\right|+C$

$56)\,\dfrac{6}{5}t^5-2t^3-3t^2+3\ln\left|t^2-t+1\right|+2\sqrt{3}\arctan\dfrac{2t-1}{\sqrt{3}}+C,\ t=\sqrt[6]{x}$

$57.\,\dfrac{1}{9}\dfrac{x}{\sqrt{x^2+9}}+C$ $58.\,\dfrac{x}{\sqrt{1-x^2}}+C$ $59.\,\dfrac{1}{16}\arctan\dfrac{x}{2}+\dfrac{1}{8}\dfrac{x}{x^2+4}+C$

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$1)\,\arctan e^x+C$ $2)\,-\dfrac{1}{\arcsin x}+C$ $3)\,\dfrac{1}{\sqrt{2}}\arcsin\left(\sqrt{\dfrac{2}{3}}\sin x\right)+C$ $4)\,\dfrac{1}{2}\ln\dfrac{x^2+1}{x^2+2}+C$

$5)\,\displaystyle\int\dfrac{\sin x\cos x}{\sqrt{a^2\sin^2 x+b^2\cos^2 x}}\,dx=C+\begin{cases}\sqrt{a^2\sin^2 x+b^2\cos^2 x}\,/(a^2-b^2)\,,&a^2\neq b^2\\[2mm]0.5(\sin x)^2/|a|\,,&a^2\neq b^2\end{cases}$

6) $\tan x - \cot x + C$ 7) $\dfrac{\ln(1+\cos^2 x) - \cos^2 x}{2} + C$ 8) $(\operatorname{arctg}\sqrt{x}\,)^2 + C$

9) $6t - 6t^2 + 2t^3 - \dfrac{3}{2}t^4 + \dfrac{5}{6}t^5 - \dfrac{6}{7}t^7 + 3\ln(1+t^2) - 6\arctan t + C, \;\; t = \sqrt[6]{x+1}$

10) $\dfrac{2x+1}{4}\sqrt{2+x+x^2} + \dfrac{7}{8}\ln\left(\dfrac{1}{2}+x+\sqrt{2+x+x^2}\right) + C$ 11) $C - \dfrac{x^2+1}{2}e^{-x^2}$

12) $\dfrac{5}{16}x + \dfrac{1}{4}\sin 2x + \dfrac{3}{64}\sin 4x - \dfrac{1}{48}\sin^3 x + C$ 13) $x - \dfrac{1}{2}\ln(1+e^{2x}) - e^{-x}\arctan e^x + C$

14) $\dfrac{1}{6}\ln\dfrac{(x+1)^2}{x^2-x+1} + \dfrac{1}{\sqrt{3}}\arctan\dfrac{2x-1}{\sqrt{3}} + C$ 15) $0.25\ln\left|\dfrac{x-1}{3x+1}\right| + C$

16) $\dfrac{1}{8}(2x^2 - 2x\sin 2x - \cos 2x) + C$ 17) $\dfrac{2x+1}{4}\sqrt{2+x-x^2} + \dfrac{9}{8}\arcsin\dfrac{2x-1}{3} + C$

18) $-\dfrac{1}{12}\arctan\dfrac{\cos^4 x}{3} + C$ 19) $\ln|1+\sin x| + C$ 20) $\dfrac{1}{8}\ln\dfrac{(1+\sin x)^2}{(1-\sin x)(3+\sin x)}$

21) $\ln|x| - \dfrac{1}{3}\ln|x^3+1| + C$ 22) $\dfrac{0.05}{2x+5} - \dfrac{0.25\ln x}{(2x+5)^2} + 0.01\ln\left|\dfrac{x}{2x+5}\right| + C$

פתרונות

13) $\left(\dfrac{1-x}{x}\right)^2 = \dfrac{1}{x^2} - \dfrac{2}{x} + 1$ 17) $x^4 + x^{-4} + 2 = (x^2 + x^{-2})^2$

18) $\dfrac{x^2}{1-x^2} = \dfrac{x^2 - 1 + 1}{1 - x^2} = -1 + \dfrac{1}{1-x^2}$ 19) $\dfrac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} = \dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1+x^2}}$

20) $(2^x + 3^x)^2 = 2^{2x} + 2 \cdot 6^x + 3^{2x}$

21) $\dfrac{2^{x+1} - 5^{x-1}}{10^x} = 2\left(\dfrac{1}{5}\right)^x - \dfrac{1}{5}\left(\dfrac{1}{2}\right)^x$ 22) $\dfrac{e^{3x}+1}{e^x+1} = \dfrac{(e^x+1)(e^{2x}-e^x+1)}{e^x+1} = e^{2x} - e^x + 1$

24) $\operatorname{ctg}^2 x = -1 + \dfrac{1}{\sin^2 x}$ 25) $\operatorname{tg}^2 x = -1 + \dfrac{1}{\cos^2 x}$ 28) $\sqrt[5]{1-2x+x^2} = (1-x)^{2/5}$

31) $1 + \cos x = 2\cos^2(x/2)$ 32) $1 - \cos x = 2\sin^2(x/2)$

33) $x(1-x)^{10} = -(-1+1-x)(1-x)^{10} = -(1-x)^{11} + (1-x)^{10}$

34) $\dfrac{x^2 - 1 + 1}{1+x} = x - 1 + \dfrac{1}{1+x}$ 35) $\dfrac{1}{\sqrt{x+1} + \sqrt{x-1}} = \dfrac{\sqrt{x+1} - \sqrt{x-1}}{(x+1) - (x-1)}$

36) $x\sqrt{2-5x} = \dfrac{1}{5}(2 - 2 + 5x)\sqrt{2-5x} = \dfrac{2}{5}\sqrt{2-5x} - \dfrac{1}{5}(2-5x)^{3/2}$

46.c) $\displaystyle\int x^2 \sqrt[3]{1+x^3}\, dx = \begin{vmatrix} 1+x^3 = t \\ 3x^2 dx = dt \end{vmatrix} = \dfrac{1}{3}\int \sqrt[3]{t}\, dt$

    d) $\displaystyle\int \dfrac{x\, dx}{3 - 2x^2}$ $3 - 2x^2 = t$ e) $\displaystyle\int \dfrac{x\, dx}{(1+x^2)^2}$ $1 + x^2 = t$

47.a) $\displaystyle\int \dfrac{x\, dx}{4 + x^4} = \dfrac{1}{2}\int \dfrac{dx^2}{4 + x^4}$, $x^2 = t$,   b) $\dfrac{x^3 dx}{x^8 - 2} = \dfrac{1}{4}\dfrac{dx^4}{(x^4)^2 - 2}$

48.a) $x\, dx = -\dfrac{1}{2}d(-x^2)$ b) $\dfrac{dx}{x^2} = -d\left(\dfrac{1}{x}\right)$ c) $x^2 dx = \dfrac{1}{2}d(x^3)$ d) $\dfrac{dx}{x} = d(\ln 5x)$

    e) $\dfrac{dx}{x\, \ln x\, \ln(\ln x)} = \dfrac{d(\ln x)}{\ln x\, \ln(\ln x)} = \big|\ln x = t\big| = \dfrac{dt}{t\, \ln(t)} = \dfrac{d(\ln t)}{\ln t} = \dfrac{du}{u}$

49.a) $\displaystyle\int \dfrac{x\, dx}{\sqrt{1-x^2}} = -\dfrac{1}{2}\int \dfrac{d(1-x^2)}{\sqrt{1-x^2}} = -\dfrac{1}{2}\int \dfrac{du}{\sqrt{u}}$

    $\displaystyle\int \dfrac{1+x}{\sqrt{1-x^2}}\, dx = \int \dfrac{dx}{\sqrt{1-x^2}} + \int \dfrac{x\, dx}{\sqrt{1-x^2}}$

$$c) \int \frac{(8x-11)\,dx}{\sqrt{5+2x-x^2}} = \begin{vmatrix} 5+2x-x^2 = -(x^2-2x-5) = \\ = -[(x-1)^2 -1 -5)] = 6-(x-1)^2 \\ x-1=t, dx=dt \end{vmatrix} = \int \frac{(8(t+1)-11)\,dt}{\sqrt{6-t^2}} =$$

$$\int \frac{8t\,dt}{\sqrt{6-t^2}} - 3\int \frac{dt}{\sqrt{6-t^2}}$$

$$57. \int \frac{dx}{(x^2+9)^{3/2}} = \begin{vmatrix} x = 3\tan t, & dx = 3dt/\cos^2 t \\ x^2+9 = 9(\tan^2 t +1) = 9/\cos^2 t \\ (x^2+9)^{3/2} = 27/\cos^3 t \end{vmatrix} = \int \frac{\cos^3 t}{27}\frac{3dt}{\cos^2 t} =$$

$$\frac{1}{9}\int \cos t\,dt = \frac{1}{9}\sin t + C = \frac{1}{9}\frac{x}{\sqrt{x^2+9}} + C$$

$$58. \int \frac{dx}{(1-x^2)^{3/2}} = \begin{vmatrix} x = \sin t, & dx = \cos t dt \\ 1-x^2 = \cos^2 t \end{vmatrix} = \int \frac{\cos t\,dt}{\cos^3 t} = \tan t + C = \frac{x}{\sqrt{1-x^2}} + C$$

$$59. \int \frac{dx}{(x^2+4)^2} = \begin{vmatrix} x = 2\tan t, & dx = 2dt/\cos^2 t \\ x^2+4 = 4(\tan^2 t +1) = 4/\cos^2 t \\ (x^2+4)^2 = 16/\cos^4 t \end{vmatrix} = \int \frac{\cos^4 t}{16}\frac{2dt}{\cos^2 t} = \frac{1}{8}\int \cos^2 t\,dt =$$

$$\frac{1}{8}\int \frac{1+\cos 2t}{2}\,dt = \frac{1}{16}t + \frac{1}{32}\sin 2t + C = \begin{vmatrix} x = 2\tan t \Rightarrow t = \arctan\frac{x}{2} \\ \sin 2t = 2\sin t\cos t = 2\frac{x}{\sqrt{x^2+4}}\frac{2}{\sqrt{x^2+4}} \end{vmatrix} =$$

$$\frac{1}{16}\arctan\frac{x}{2} + \frac{1}{8}\frac{x}{x^2+4} + C$$

תרגילי אינטגרציה שונים

$$1. \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x dx}{e^{2x}+1} = \int \frac{d(e^x)}{e^{2x}+1} = \arctan e^x + C$$

$$3. \int \frac{\cos x\,dx}{\sqrt{2+\cos 2x}} = \frac{1}{\sqrt{2}}\int \frac{d(\sqrt{2}\sin x)}{\sqrt{3-2\sin^2 x}} = \frac{1}{\sqrt{2}}\arcsin\left(\sqrt{\frac{2}{3}}\sin x\right) + C$$

$$5. \int \frac{\sin x\cos x}{\sqrt{a^2\sin^2 x + b^2\cos^2 x}}\,dx \qquad a^2\sin^2 x + b^2\cos^2 x = t$$

18) $\int \dfrac{\sin x \cos^3 x}{9 + \cos^8 x}\,dx = -\int \dfrac{\cos^3 x}{9 + \cos^8 x}\,d(\cos x) = -\int \dfrac{z^3}{9 + z^8}\,dz = -\dfrac{1}{4}\int \dfrac{dz^4}{9 + z^8} = -\dfrac{1}{4}\int \dfrac{dt}{9 + t^2} =$

$-\dfrac{1}{12}\arctan \dfrac{\cos^4 x}{3} + C$

19) $\int \dfrac{\cos x - \sin x + 1}{\cos x + \sin x + 1}\,dx = \int \dfrac{2\cos^2 \dfrac{x}{2} - 2\sin \dfrac{x}{2}\cos \dfrac{x}{2}}{2\cos^2 \dfrac{x}{2} + 2\sin \dfrac{x}{2}\cos \dfrac{x}{2}}\,dx = \int \dfrac{\cos \dfrac{x}{2} - \sin \dfrac{x}{2}}{\cos \dfrac{x}{2} + \sin \dfrac{x}{2}}\,dx =$

$= \int \dfrac{2d\left(\cos \dfrac{x}{2} + \sin \dfrac{x}{2}\right)}{\cos \dfrac{x}{2} + \sin \dfrac{x}{2}} = 2\ln\left|\cos \dfrac{x}{2} + \sin \dfrac{x}{2}\right| + C = \ln\left|1 + \sin x\right| + C$