# אינטגרל בלתי מסוים

13) $\left(\frac{1-x}{x}\right)^2 = \frac{1}{x^2} - \frac{2}{x} + 1$ 17) $x^4 + x^{-4} + 2 = (x^2 + x^{-2})^2$

18) $\frac{x^2}{1-x^2} = \frac{x^2 - 1 + 1}{1-x^2} = -1 + \frac{1}{1-x^2}$ 19) $\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1+x^2}}$

20) $(2^x + 3^x)^2 = 2^{2x} + 2 \cdot 6^x + 3^{2x}$

21) $\frac{2^{x+1} - 5^{x-1}}{10^x} = 2\left(\frac{1}{5}\right)^x - \frac{1}{5}\left(\frac{1}{2}\right)^x$ 22) $\frac{e^{3x}+1}{e^x+1} = \frac{(e^x+1)(e^{2x} - e^x + 1)}{e^x+1} = e^{2x} - e^x + 1$

24) $\operatorname{ctg}^2 x = -1 + \frac{1}{\sin^2 x}$ 25) $\operatorname{tg}^2 x = -1 + \frac{1}{\cos^2 x}$ 28) $\sqrt[5]{1 - 2x + x^2} = (1-x)^{2/5}$

31) $1 + \cos x = 2\cos^2(x/2)$ 32) $1 - \cos x = 2\sin^2(x/2)$

33) $x(1-x)^{10} = -(-1+1-x)(1-x)^{10} = -(1-x)^{11} + (1-x)^{10}$

34) $\frac{x^2 - 1 + 1}{1+x} = x - 1 + \frac{1}{1+x}$ 35) $\frac{1}{\sqrt{x+1} + \sqrt{x-1}} = \frac{\sqrt{x+1} - \sqrt{x-1}}{(x+1) - (x-1)}$

36) $x\sqrt{2-5x} = \frac{1}{5}(2 - 2 + 5x)\sqrt{2-5x} = \frac{2}{5}\sqrt{2-5x} - \frac{1}{5}(2-5x)^{3/2}$

46.c) $\int x^2 \sqrt[3]{1+x^3}\,dx = \left|\begin{matrix} 1+x^3 = t \\ 3x^2 dx = dt \end{matrix}\right| = \frac{1}{3}\int \sqrt[3]{t}\,dt$

d) $\int \frac{x\,dx}{3-2x^2}$ $3 - 2x^2 = t$ e) $\int \frac{x\,dx}{(1+x^2)^2}$ $1 + x^2 = t$

47.a) $\int \frac{x\,dx}{4+x^4} = \frac{1}{2}\int \frac{dx^2}{4+x^4}$, $x^2 = t$, b) $\frac{x^3 dx}{x^8 - 2} = \frac{1}{4}\frac{dx^4}{(x^4)^2 - 2}$

48.a) $xdx = -\frac{1}{2}d(-x^2)$ b) $\frac{dx}{x^2} = -d\left(\frac{1}{x}\right)$ c) $x^2 dx = \frac{1}{2}d(x^3)$ d) $\frac{dx}{x} = d(\ln 5x)$

e) $\frac{dx}{x\ \ln x \ln(\ln x)} = \frac{d(\ln x)}{\ln x \ln(\ln x)} = \left|\ln x = t\right| = \frac{dt}{t\ln(t)} = \frac{d(\ln t)}{\ln t} = \frac{du}{u}$

49.a) $\int \frac{xdx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{d(1-x^2)}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{du}{\sqrt{u}}$

$\int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{xdx}{\sqrt{1-x^2}}$

$$c)\int\frac{(8x-11)\,dx}{\sqrt{5+2x-x^2}}=\left|\begin{array}{l}5+2x-x^2=-(x^2-2x-5)=\\=-[(x-1)^2-1-5)]=6-(x-1)^2\\x-1=t,dx=dt\end{array}\right|=\int\frac{(8(t+1)-11)\,dt}{\sqrt{6-t^2}}=$$

$$\int\frac{8t\,dt}{\sqrt{6-t^2}}-3\int\frac{dt}{\sqrt{6-t^2}}$$

$$57.\int\frac{dx}{(x^2+9)^{3/2}}=\left|\begin{array}{l}x=3\tan t,\quad dx=3dt/\cos^2 t\\x^2+9=9(\tan^2 t+1)=9/\cos^2 t\\(x^2+9)^{3/2}=27/\cos^3 t\end{array}\right|=\int\frac{\cos^3 t}{27}\frac{3dt}{\cos^2 t}=$$

$$\frac{1}{9}\int\cos t\;dt=\frac{1}{9}\sin t+C=\frac{1}{9}\frac{x}{\sqrt{x^2+9}}+C$$

$$58.\int\frac{dx}{(1-x^2)^{3/2}}=\left|\begin{array}{l}x=\sin t,\quad dx=\cos t dt\\1-x^2=\cos^2 t\end{array}\right|=\int\frac{\cos t\;dt}{\cos^3 t}=\tan t+C=\frac{x}{\sqrt{1-x^2}}+C$$

$$59.\int\frac{dx}{(x^2+4)^2}=\left|\begin{array}{l}x=2\tan t,\quad dx=2dt/\cos^2 t\\x^2+4=4(\tan^2 t+1)=4/\cos^2 t\\(x^2+4)^2=16/\cos^4 t\end{array}\right|=\int\frac{\cos^4 t}{16}\frac{2dt}{\cos^2 t}=\frac{1}{8}\int\cos^2 t\;dt=$$

$$\frac{1}{8}\int\frac{1+\cos 2t}{2}\,dt=\frac{1}{16}t+\frac{1}{32}\sin 2t+C=\left|\begin{array}{l}x=2\tan t\Rightarrow t=\arctan\dfrac{x}{2}\\\sin 2t=2\sin t\cos t=2\dfrac{x}{\sqrt{x^2+4}}\dfrac{2}{\sqrt{x^2+4}}\end{array}\right|=$$

$$\frac{1}{16}\arctan\frac{x}{2}+\frac{1}{8}\frac{x}{x^2+4}+C$$

## תרגילי אינטגרציה שונים

$$1.\int\frac{dx}{e^x+e^{-x}}=\int\frac{e^x dx}{e^{2x}+1}=\int\frac{d(e^x)}{e^{2x}+1}=\arctan e^x+C$$

$$3.\int\frac{\cos x dx}{\sqrt{2+\cos 2x}}=\frac{1}{\sqrt{2}}\int\frac{d(\sqrt{2}\sin x)}{\sqrt{3-2\sin^2 x}}=\frac{1}{\sqrt{2}}\arcsin\left(\sqrt{\frac{2}{3}}\sin x\right)+C$$

$$5.\int\frac{\sin x\cos x}{\sqrt{a^2\sin^2 x+b^2\cos^2 x}}dx\qquad a^2\sin^2 x+b^2\cos^2 x=t$$

18) $\int \frac{\sin x \cos^3 x}{9+\cos^8 x} dx = -\int \frac{\cos^3 x}{9+\cos^8 x} d(\cos x) = -\int \frac{z^3}{9+z^8} dz = -\frac{1}{4}\int \frac{dz^4}{9+z^8} = -\frac{1}{4}\int \frac{dt}{9+t^2} =$

$-\frac{1}{12}\arctan\frac{\cos^4 x}{3} + C$

19) $\int \frac{\cos x - \sin x + 1}{\cos x + \sin x + 1} dx = \int \frac{2\cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}} dx = \int \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} dx =$

$= \int \frac{2d\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\cos\frac{x}{2} + \sin\frac{x}{2}} = 2\ln\left|\cos\frac{x}{2} + \sin\frac{x}{2}\right| + C = \ln|1+\sin x| + C$