1. Calculus Gimmel 2 - Exercise 7a

1.1. Formula for surface area.

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

1.2. Find the area of the surface.

(1) The part of the plane $z = 2 + 3x + 4y$ that lies above the rectangle $[0, 5] \times [1, 4]$.

(2) The part of the plane $2x+5y+z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$.

(3) The part of the plane $3x + 2y + z = 6$ that lies in the first octant.

(4) The part of the surface $z = 1+3x+2y^2$ that lies above the triangle with vertices $(0, 0), (0, 1),$ and $(2, 1)$.

(5) The part of the cylinder $y^2+z^2 = 9$ that lies above the rectangle with vertices $(0, 0), (4, 0), (0, 2),$ and $(4, 2)$.

(6) The part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.

(7) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(8) The surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$, $0 \leq x \leq 1$, $0 \leq y \leq 1$.

(9) The part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

(10) The part of the sphere $x^2+y^2+z^2 = 4$ that lies above the plane $z = 1$.

(11) The part of the sphere $x^2 + y^2 + z^2 = a^2$, that lies within the cylinder $x^2 + y^2 = ax$, and above the xy-plane.

(12) The part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside the paraboloid $z = x^2 + y^2$.

1.3. Partial answers. (1).$15\sqrt{26}$, (3).$3\sqrt{14}$, (5).$12 \arcsin(\frac{2}{3})$, (7).$\frac{\pi}{6}(17\sqrt{17} - 5\sqrt{5})$, (9).$\frac{2\pi}{3}(2\sqrt{2} - 1)$, (11).$a^2(\pi - 2)$.

1.4. **Partial solution.** (7). The surface area formula gives
$A(S) = \iint_D \sqrt{1 + (-2x)^2 + (2y)^2}\, dA = \iint_D \sqrt{1 + 4(x^2 + y^2)}\, dA.$
Converting to polar coordinates, we get

$$\begin{aligned}
\int_0^{2\pi} \int_1^2 \sqrt{1 + 4r^2} r \, dr \, d\theta &= \frac{1}{8} \int_0^{2\pi} d\theta \int_1^2 \sqrt{1 + 4r^2}\, d(1 + 4r^2) \\
&= \frac{1}{8} \cdot 2\pi \cdot \frac{2}{3} (1 + 4r^2)^{3/2}|_1^2 \\
&= \frac{\pi}{6} \big((1 + 16)^{3/2} - (1 + 4)^{3/2} \big) \\
&= \frac{\pi}{6} \big(17\sqrt{17} - 5\sqrt{5} \big).
\end{aligned}$$