Numerical methods using Matlab, 001-2-7011: Assignment 2

1. A ball of radius *R*, made of material with the density ρ_s, is placed into a liquid of density ρ_l, $0<\rho_s<\rho_l$. It is needed to calculate the depth h to which the ball is submerged. Derive an equation $F(x,r)=0$ determining $x=h/R$ for a given ratio $r=\rho_s/\rho_l$. Write an m-function x=depth(r,tol) that uses Newton method to solve this equation with a given tolerance, $|F(x,r)|$ < tol, for a given density ratio r. Explain your choice of the initial approximation x_0. Plot and attach to your report a graph $x=x(r)$ for $0\le r\le 1$.
2. Suppose your computer forgot how to calculate logarithms and you need to write your own m-function y=my_log(x,abs_err) to compute natural logarithms $y=\ln(x)$ for $x>0$ with the absolute error not exceeding abs_err.
   A possible algorithm is based on the Taylor expansion of logarithm,
   $$\ln(1-z)=-z-\frac{z^2}{2}-\frac{z^3}{3}-\ldots$$
   The series converges only if $|z|<1$ and the convergence is very slow for z close to ± 1. You can circumvent this limitation as follows. Let the value of e be known. If $x>1$ we find the minimal integer m such that $x<e^m$ and use the equality
   $\ln(x)=m+\ln(xe^{-m})$. Then $\frac{1}{e}<xe^{-m}=1-z$, where $0<z<1-\frac{1}{e}\approx 0.63$ and the series converges quickly. To decide how many terms of the series to sum up (you need to estimate the absolute error) note that
   $$\sum_{k=N}^{\infty}\frac{z^k}{k}<\frac{z^N}{N}\left(1+z+z^2+\ldots\right)=\frac{z^N}{N(1-z)}.$$
   Finally, for 0< $x<1$ we have $\ln(x)=-\ln(1/x)$, so the same algorithm can be used.

3. Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance one can determine the temperature. Steinhart-Hart equation for the 10k3A Betathern thermistor is (A. Kaw)
   $$\frac{1}{T}=a_0+a_1\ln(R)+a_2\ln^3(R),$$
   where *T* is in Kelvin, *R* is in Ohms, $a_0=1.12924\times 10^{-3}$, $a_1=2.341077\times 10^{-3}$, $a_2=8.775468\times 10^{-8}$. Suppose that $1\le R\le 5$ and is measured with the relative error $\delta(R)\le 0.001$. To calculate $\ln(R)$ you use your program my_log. Is it possible to find *T* with the absolute

error $\Delta(T) \le 2^o K$? If possible, what should be the absolute error of $\ln(R)$ calculation specified in my_log? Explain!

4. Write an m-function u=Newton2(F,u0,tol,maxit) to solve systems of two nonlinear equations

$$\begin{cases} f_1(x,y)=0 \\ f_2(x,y)=0 \end{cases}$$

by the 2D Newton method (copy and use the file Newton2.m; it contains this function specification). As an example, use this program to find all solutions of

$$\begin{cases} x^5 - x^3 - y^5 + xy + 0.1 = 0 \\ x^4 + y^4 - 1 + 0.5\sin(8xy) = 0 \end{cases}$$

Hint: To find a good initial approximation to each solution you can use Matlab function contour to find points where $|f_1|+|f_2|$ is small, and Matlab function ginput to obtain their coordinates interactively. It is possible also to plot zero contours of both functions and use ginput to obtain the approximate coordinates of these contours intersections.