

תרגילים נוספים למשוואת החום

1) $u_t = 4u_{xx} - u + t$, $u(0,t) = t$, $u(\pi,t) = t+3$, $t > 0$, $u(x,0) = 3$, $0 < x < \pi$

2) $u_t = u_{xx} - 2u + x(1+2t) + 5 \sin 2t \cos x e^{-2t}$
 $u_x(0,t) = t$, $u(\pi/2,t) = \pi t/2$, $t > 0$, $u(x,0) = \begin{cases} 1, & 0 \leq x \leq \pi/4 \\ 0, & \pi/4 < x < \pi/2 \end{cases}$

3) $u_t = 9u_{xx} - u + (2 \sin \pi x + 9\pi^2 t \sin 3\pi x + 2)e^{-t}$
 $u(0,t) = (2t+1)e^{-t}$, $u_x(1/2,t) = 0$, $t > 0$, $u(x,0) = x+1$, $0 \leq x \leq 1/2$

4) $u_t = \pi u_{xx} - 3u - 2\pi + 3x^2 + e^{-3t} \sin \pi x$
 $u_x(0,t) = e^{-3t}$, $u_x(\pi,t) = 2\pi + e^{-3t}$, $t > 0$, $u(x,0) = x^2 + 2x$, $0 < x < \pi$

5) $u_t = 4u_{xx} - 3u + 12x + 2e^{-t} + te^{-3t} \sin 2\pi x$
 $u(0,t) = e^{-t}$, $u(1,t) = e^{-t} + 4$, $t > 0$, $u(x,0) = 1+4x-5 \sin 3\pi x$, $0 < x < 1$

6) $u_t = 9u_{xx} - u + 1$, $u_x(0,t) = 0$, $u(\pi,t) = t$, $t > 0$, $u(x,0) = \cos(3x/2)$, $0 < x < \pi$

7) $u_t = 3u_{xx} - 2u + \gamma(2x - 2 + \cos 2x)$, $\gamma = const$
 $u_x(0,t) = u_x(\pi,t) = \gamma$, $t > 0$, $u(x,0) = \gamma(x + 2 \cos^2 x)$, $0 < x < \pi$

תשובות:

1) $u(x,t) = t + 3x/\pi + \sum_{n=1}^{\infty} \left[\frac{\beta_n}{4n^2 + 1} + \left(\frac{6}{n\pi} - \frac{\beta_n}{4n^2 + 1} \right) e^{-(4n^2+1)t} \right] \sin nx$, $\beta_n = 2 \frac{4(-1)^n - 1}{n\pi}$

2) $u(x,t) = tx + \left(2(1 + \sqrt{2}/\pi)e^{-3t} + (\sin 2t - 2 \cos 2t)e^{-2t} \right) \cos x +$
 $+ \frac{4}{\pi} \sum_{n=2}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi}{4} e^{-(2n-1)^2 t} \cos(2n-1)x$

3) $u(x,t) = e^{-t} \left[2t + 1 + \frac{1}{9\pi^2} \left((34e^{-9\pi^2 t} + 2) \sin \pi x + \left(\left(\frac{1}{81} - 4 \right) e^{-81\pi^2 t} + \frac{81\pi^2 t - 1}{81} \right) \sin 3\pi x \right) \right] +$
 $+ e^{-t} \frac{4}{\pi^2} \sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} e^{-9(2n-1)^2 \pi^2 t} \sin(2n-1)\pi x$

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$$4) u(x, t) = x^2 + xe^{-3t} + e^{-3t} \left[\frac{2t}{\pi} + \frac{1}{2} - \frac{4}{\pi^2} e^{-\pi^3 t} \cos \pi x + \sum_{n=2}^{\infty} w_n \cos n\pi x \right]$$

$$w_n = \left(\gamma_n - \frac{\beta_n}{n^2 \pi^3} \right) e^{-n^2 \pi^3 t} + \frac{\beta_n}{n^2 \pi^3}, n = 2, 3, \dots$$

$$\beta_n = -2 \frac{(-1)^n + 1}{(n^2 - 1)} = \begin{cases} -4 & , n = 2k \\ 0 & , n = 2k + 1 \end{cases} \Rightarrow w_n = \begin{cases} \frac{\beta_n}{n^2 \pi^3} (1 - e^{-n^2 \pi^3 t}), & n = 2k \\ \gamma_n e^{-n^2 \pi^3 t} & , n = 2k + 1 \end{cases}$$

$$\gamma_n = 2 \frac{(-1)^n - 1}{n^2 \pi^2} = \begin{cases} -4 & , n = 2k + 1 \\ 0 & , n = 2k \end{cases}$$

$$w_n = \frac{-4}{\pi^2} \begin{cases} \frac{1}{n^2 (n^2 - 1) \pi} (1 - e^{-n^2 \pi^3 t}) & , n = 2k \\ \frac{1}{n^2} e^{-n^2 \pi^3 t} & , n = 2k + 1 \end{cases}, k = 1, 2, \dots$$

$$5) u(x, t) = 4x + e^{-t} + e^{-3t} \frac{e^{-16\pi^2 t} + 16\pi^2 t - 1}{256\pi^4} \sin 2\pi x - 5e^{-(36\pi^2 + 3)t} \sin 3\pi x$$

$$6) u(x, t) = t + \sum_{n=1}^{\infty} \left[C_n e^{-\gamma_n t} + \frac{\beta_n}{\gamma_n} \left(t - \frac{1}{\gamma_n} \right) \right] \cos \frac{2n-1}{2} x$$

$$\gamma_n = 1 + \frac{9(2n-1)^2}{4}, \quad \beta_n = \frac{4(-1)^n}{\pi(2n-1)}, \quad C_n = \frac{\beta_n}{\gamma_n^2} + \begin{cases} 1, & n = 2 \\ 0, & n \neq 2 \end{cases}$$

$$7) u(x, t) = \gamma \left(x + (2e^{-2t} - 1) + \frac{1}{14} (13e^{-14t} + 1) \cos 2x \right)$$