ADVANCED ANALYSIS (201-2-5401)
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HOMEWORK ASSIGNMENT 6

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We use $\mathcal{K}$ to denote $\mathcal{K}(H)$ or $\mathcal{K}(E, F)$ — the relevant space of compact operators. H is always a Hilbert space. Do not hand in.

Exercise 1. Prove:

(1) Every finite rank operator is compact (T is called *a finite rank operator* if it is bounded and the dimension of its rank is finite).
(2) $\mathcal{K}(E, F)$ is a closed subspace of $B(E, F)$.
(3) If $A \in B(F, G), B \in B(D, E)$ and $K \in \mathcal{K}(E, F)$, then $AKB \in \mathcal{K}(D, G)$.

Exercise 2. Let $k \in C([a, b] \times [a, b])$ and define an operator $T : C([a, b]) \to C([a, b])$ by

$$Tf(x) = \int_0^1 k(x, t) f(t) dt.$$

Prove that T is compact. One can also define an operator $S : L^2([a, b]) \to L^2([a, b])$ by the same formula. Is S also compact?

Exercise 3. Let U be the bilateral shift on $\ell^2(\mathbb{Z})$:

$$Ux = y$$

where $y_n = x_{n-1}$. U is normal. Find the representation of U as a multiplication operator.