C^*-ALGEBRAS ARISING FROM SUBPRODUCT SYSTEMS
ABSTRACT

Subproduct systems were introduced for the purpose of studying quantum dynamical semigroups. In the two short years since their introduction there has been growing interest in subproduct systems, their applications to quantum dynamics and the operator algebras that they give rise to. Several different operator algebras of interest can be associated even with the simplest classes of subproduct systems. Until now, most of the attention has been devoted to the *non-selfadjoint* operator algebras arising from subproduct systems.

The aim of the proposed project is to study several kinds of operator algebras, with an emphasis on C*-algebras, which arise from subproduct systems, and to understand their structures, their inter-relations, and the way in which they reflect the subproduct system and the underlying dynamics. For a special class of subproduct systems — *commutative* subproduct systems — the success of this project will touch upon a far–reaching conjecture of Arveson that connects between commutative and noncommutative topology.

A *subproduct system* is a family $X = \{X(n)\}_{n\in\mathbb{N}}$ of finite dimensional Hilbert spaces such that

$$X(m+n) \subseteq X(m) \otimes X(n) \ , \ m,n \in \mathbb{N}.$$

With every subproduct system, one can naturally associate the *tensor algebra* $\mathcal{A}_X$, which is a non-selfadjoint operator algebra arising from the subproduct system structure. The *Toeplitz algebra of* X, denoted $\mathcal{T}_X$, is defined as the C*-algebra generated by $\mathcal{A}_X$, and the *Cuntz algebra of* X, denoted $\mathcal{O}_X$, is the quotient of $\mathcal{T}_X$ by the compacts. Finally, there is also the C*-envelope of $\mathcal{A}_X$, which we denote by $C^*_e(\mathcal{A}_X)$. The collection of algebras arising in these ways contain very wide classes of operator algebras, including: the classical Toeplitz algebra, the function algebras $A(\mathbb{D})$, $C(\mathbb{T})$ and $C(\partial\mathbb{B}_d)$, the Cuntz algebra, the noncommutative disc algebra, Cuntz-Krieger algebras, subshift C*-algebras, universal operator algebras for polynomial relations, and many interesting algebras of bounded holomorphic functions on algebraic varieties.

The two main problems I propose to address are:

(1) To compute the C*-envelopes of the algebras $\mathcal{A}_X$. Specifically, I wish to understand the boundary representations of $\mathcal{A}_X$ relative to $\mathcal{T}_X$, to identify the Shilov ideal, and to understand the manner in which X determines the structure of $C^*_e(\mathcal{A}_X)$. This will shed light on the connections between the different algebras mentioned above.

(2) When X is a commutative subproduct system, then one may naturally associate with it a homogeneous algebraic variety $V(X)$. A conjecture of Arveson states that $\mathcal{O}_X$ is commutative, and it therefore implies that $\mathcal{O}_X = C(V(X)\cap\partial\mathbb{B}_d)$. I propose to study the easier problem of whether or not $\mathcal{O}_X$ is a topological invariant for $V(X)$, and if so whether or not it is a complete invariant.

Both problems present major challenges. But in light of the many connections to current research areas, as well as the intrinsic beauty of the problems at hand, one is compelled to confront these challenges.