Scientific abstract

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OPERATOR-ALGEBRAIC GEOMETRY IN THE DRURY-ARVESON HILBERT MODULE

The Drury-Arveson Hilbert module H^2_d plays a key role in multivariable operator theory, and constitutes a junction where various branches of mathematics such as operator theory, operator algebras, function theory in the unit ball of $\mathbb{C}^d$, complex and algebraic geometry, harmonic analysis and K-homology all meet.

Inspired by classical algebraic geometry, the proposed research is aimed at using the geometry of varieties in the unit ball $\mathbb{B}_d$ to study the operator-algebraic structures associated with H^2_d. Denote by $\mathcal{M}_d$ the multiplier algebra of H^2_d. For an analytic variety $V \subset \mathbb{B}_d$, let $J_V \lhd \mathcal{M}_d$ be the ideal of functions that vanish on V. I will focus on the following three concrete problems:

(1) The quotient algebra $\mathcal{M}_d/J_V$ can be identified with the algebra of restrictions to V, i.e., $\mathcal{M}_V := \{f\big|_V : f \in \mathcal{M}_d\}$. How does the geometry of V determine the structure of this algebra? Roughly, I conjecture that for varieties V and W, $\mathcal{M}_V$ and $\mathcal{M}_W$ are isomorphic if and only if V and W are biholomorphic in a strong sense.

(2) In the case where V is a homogeneous algebraic variety, I seek an effective version of Hilbert's Basis Theorem: namely a basis $f_1, \ldots, f_k$ of polynomials in J_V such that for every polynomial h in J_V, one can find polynomials $g_1, \ldots, g_k$ such that $h = \sum g_i f_i$, with some norm control, for example:

$$\sum \|g_i f_i\| \leq C\|h\|.$$

(3) Let $S_1, \ldots, S_d$ denote the natural shift on H^2_d restricted to $\overline{J_V}$. For V as above, I aim to prove the following special case of Arveson's Conjecture: *for all i, j the commutant $S_i S_j^* - S_j^* S_i$ is in the Schatten p-class for all $p > d$. In particular, it is compact.*

These are central and fundamental problems in multivariable operator theory. Progress made on any one of these problems will not only be of great theoretical interest to the research community, but will necessitate the development of new techniques which could find applications elsewhere.