Combinatorial Geometry: Exercise 3.
Due date: 16.12.2010

1. (i) Prove a "similar" version of the week cutting lemma for circles.

   (ii) Use (i) to prove that the number of incidences between n points and n (arbitrary) circles is $O(n^{1.4} \log^c n)$ (for some constant c).

2. Let $\mathcal{H}$ be a set of n hyperplanes in $\mathbb{R}^d$. For two points $p, q \in \mathbb{R}^d$ we let $d_{\mathcal{H}}(p,q)$ be the number of hyperplanes in H that are intersected by the interior of the line segment pq. Show that there exists an absolute constant C such that for every parameter $1 \leq r \leq n$ there exists a subset $\mathcal{H}' \subset \mathcal{H}$ of size at most $Cdr \log n$ such that if $d_{\mathcal{H}}(p,q) \geq \frac{n}{r}$ then $d_{\mathcal{H}'}(p,q) \geq 1$ for any pair of points $p, q \in \mathbb{R}^d$.

3. Let P be a set of n points such that no three of them lie on the same line and no two points have the same x-coordinate. Let $1 \leq k \leq n$ be a given parameter. Let E be the set of all pairs $p, q \in P$ such that the line $\ell_{p,q}$ passing through p and q has at most k points of P that lie completely below it. Prove that for the graph $G = (P, E)$ we have $\chi(G) = O(k)$.