

Combinatorial Geometry: Exercise 4.

Due date: 6/01/2011

1. Prove that for every n and s , $\lambda_s(n) \leq 1 + (s + 1) \binom{n}{2}$
2. Let L be a family of n lines in the plane. For an additional line l' , define the *zone* of l' to be the subset of cells in the arrangement $\mathcal{A}(L)$ intersected by the line l' . The complexity of the zone is the sum of sizes of these cells. Prove that the zone of any line l' has complexity $O(n)$. Hint: Consider only one side of l' and denote each side of each line in L with a distinct symbol. Use the bound $\lambda_2(n) = O(n)$.

3. Without using duality of LP, prove that for any hypergraph $H = (V, E)$ (with V finite) we have:

$$\nu^*(H) \leq \tau^*(H)$$

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4. Prove that there exists an absolute constant C such that any set of n points on the line can be colored with at most $Ck \log n$ colors such that any interval that contains at least k of the points must contain at least k points whose colors are all unique in that interval.