Graph Theory: Homework 1. Due Nov 21st 2016

1. Prove that for every graph G either G or $\bar{G}$ is connected.

2. Prove that every set of six people contains (at least) three mutual acquaintances or three mutual strangers. Solve by phrasing the problem in a graph theoretical way.

3. Let G be a graph with girth 5. Prove that if $\delta(G) \geq k$ then G has at least k^2+1 vertices. For $k=2$ and $k=3$, find one such graph with exactly k^2+1 vertices.

4. A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list: (i) Can the Peterson graph be decomposed into 3 paths of length 5?(ii) Prove that a graph with more than six vertices of odd degree cannot be decomposed into three paths.

5. prove that every graph G contains a bipartite graph H such that $e(H) \geq \frac{e(G)}{2}$.

6. A *strip* is the area in the plane enclosed between some two parallel lines. An *axis-parallel strip* is a strip whose bounding lines are parallel to one of the axes. Let P be a set of $2n$ points in the plane (for some integer n). Prove that the points of P can be colored with 'red' or 'blue' such that in any axis-parallel strip the difference between the number of red points and the number of blue points is at most 2. Hint: Construct a graph on the points such that there is an edge $\{p_{2i-1}, p_{2i}\}$ and $\{p_{\pi_{2i-1}}, p_{\pi_{2i}}\}$ for $i = 1, \ldots, n$ where $(p_1, \ldots, p_{2n})$ is the non-decreasing order of the points according to their x-coordinates and $(p_{\pi_1}, \ldots, p_{\pi_{2n}})$ is the non-decreasing order of the points according to their y-coordinates.