

Graph Theory: Homework 6, Due 10.01.2012

1. Let G be a graph such that $\chi'(G) = 2$. Prove that:
 - (a) $ch(G) = 2$
 - (b) $ch'(G) = 2$
2. Given a set of lines in the plane with no three meeting at a common point, and a circle that contain all intersection points of the lines in its interior, form a graph G whose vertices are the intersections of the pair of lines and the intersections of the lines with the circle with two vertices adjacent if they appear consecutive on one of the lines or consecutive on the circle. Prove that $\chi(G) \leq 3$.
3. Let $G = (V, E)$ be an even cycle. Let $\{L_x\}_{x \in V \cup E}$ be a family of sets, each of cardinality 4. Prove that there is a function $f : V \cup E \rightarrow \bigcup_{x \in V \cup E} L_x$ such that:
 - (a) $\forall x \in V \cup E f(x) \in L_x$
 - (b) $f|_V$ is a proper vertex coloring of G and $f|_E$ is a proper edge coloring of G .
 - (c) $\forall v \in V e \in E$ if $v \in e$ then $f(v) \neq f(e)$.
4. Prove that for every graph $G = (V, E)$ with n vertices $ch(G) < \chi(G) \ln n + 1$.