A graph theoretic approach to graded identities on matrices

This work is joint with Michael Natapov. If G is a finite group of order k we may imbed G into the group of permutation matrices in $M_k(\mathbb{C})$ and regard $M_k(\mathbb{C})$ as a graded G–algebra by setting $M_k(\mathbb{C})_g = DP_g$ where D is the set of diagonal matrices and P_g is the permutation matrix corresponding to g in G. In this lecture I will discuss the G–graded polynomial identities of $M_k(\mathbb{C})$, that is, the polynomials in noncommuting variables $x_{i,g}$ $(i \geq 1, g \in G)$ that vanish for all homogeneous substitutions. Our approach is to associate a certain finite directed graph to each monomial $x_{1,g_1}x_{2,g_2}\cdots x_{n,g_n}$. It turns out that for any permutation π the difference $x_{1,g_1}x_{2,g_2}\cdots x_{n,g_n} - x_{\pi(1),g_{\pi(1)}}x_{\pi(2),g_{\pi(2)}}\cdots x_{\pi(n),g_{\pi(n)}}$ is a graded identity if and only if the two monomials determine the same graph and such differences generate the T-ideal of graded identities. The use of these graphs greatly simplifies and clarifies several known results in the theory of graded identities. Our most substantial new result is the determination of the asymptotic behavior for the codimension of graded identities.