FINITE-DIMENSIONAL CONTRACTIVE REALIZATIONS ON POLYNOMIALLY DEFINED DOMAINS AND DETERMINANTAL REPRESENTATIONS OF MULTIVARIABLE POLYNOMIALS

DMITRY KALIUZHNYI-VERBOVETSKYI

We prove that a matrix-valued rational function F which is regular on a domain in $\mathbb{C}^d$ defined by the inequalities $\|P_i(z)\| < 1$, $i = 1, \ldots, k$, where P_i are matrix polynomials, and which has the associated Agler norm strictly less than 1, admits a finite-dimensional contractive realization

$$F(z) = D + C\mathbf{P}(z)(I - A\mathbf{P}(z))^{-1}B,$$

where $\mathbf{P}(z)$ is a direct sum of blocks $P_i(z) \otimes I_{n_i}$. As a consequence, we show that any polynomial with no zeros on the domain closure is a factor of $\det(I - K\mathbf{P}(z))$, for some strictly contractive matrix K. In the case where the domain is a matrix unit ball, we show that, in fact, a power of the polynomial admits the above determinantal representation. The latter result is obtained via noncommutative lifting and a theorem on singularities of minimal noncommutative full-structured system realizations.

The talk is based on a current joint project with A. Grinshpan, V. Vinnikov, and H. J. Woerdeman.