Introduction to Probability

Exercise sheet 3+ (additional exercises)

Exercise 1. Let X be a discrete random variable with density

$$f_X(k) = \frac{10-k}{40} \qquad k = 0, 1, 2, 3, 4,$$

and $f_X(r) = 0$ otherwise.

Show that f_X is indeed a density. What is the distribution function F_X? Calculate $\mathbb{P}[1 \leq X \leq 3]$.

Exercise 2. Let X be a random variable with distribution function

$$F_X(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{8} & 0 \leq t < 1 \\ \frac{3}{8} & 1 \leq t < 2 \\ \frac{3}{4} & 2 \leq t < 3 \\ 1 & t \geq 3. \end{cases}$$

Show that X is discrete.

What is the density of X?

Exercise 3. There are 3 chemists and 5 biologists. Out of these, 5 people are chosen, all choices equally likely. Let X be the number of chemists chosen. What is the density of X?

Exercise 4. The number of calls to the call center in an hour has Poi(20) distribution. What is the probability that there are no calls during the hour?

Exercise 5. X is an absolutely continuous random variable with density

$$f_X(s) = \begin{cases} cs^2 & 0 < s < 5 \\ 0 & \text{otherwise.} \end{cases}$$

What is c? What is the distribution function of X? Find t such that $\mathbb{P}[X < t] = 1/3$.

Exercise 6. Let $X \sim \text{Exp}(5)$. Find t so that $\mathbb{P}[X > t] = e^{-1}$.

Exercise 7. Let $X \sim U[0, 2\pi]$. Calculate $\mathbb{P}[\cos X > 0]$.

Exercise 8. Let $X \sim \text{Exp}(\lambda)$. Calculate $\mathbb{P}[\sin X > 0]$.

Exercise 9. Let $X \sim \text{Exp}(\lambda)$. Let $Y = \lfloor X \rfloor$ (the largest integer that is at most X). Show that Y is discrete. What is the density of Y?

Exercise 10. Let X be an absolutely continuous random variable with density $f_X(t) = 2te^{-t^2}$ for $t \geq 0$ and $f_X(t) = 0$ for $t < 0$. Let $Y = X^2$. Show that Y is an absolutely continuous random variable. What is the density?

Exercise 11. Let $X \sim N(0, 1)$. Let

$$Y = \text{sign}(X) = \begin{cases} 1 & X > 0 \\ 0 & X = 0 \\ -1 & X < 0. \end{cases}$$

What is the distribution of Y? Show that Y is discrete and compute the density of Y.

Let $Z = (Y + 1)/2$. What is the distribution of Z?