

Introduction to Probability

Exercise sheet 3

Exercise 1. 5 cards labelled 10, J , Q , K , A are ordered randomly, all orders equally likely. What is the probability that A is first and K is fourth, *conditioned on* Q being third?

Exercise 2. In Eurasia 10% of males are illiterate, and 5% of females are illiterate. The population consists of 40% males and 60% females. A person is chosen at random, all people equally likely.

- (a) What is the probability that that person is illiterate?
- (b) What is the probability that the person is a male given that they are illiterate?

Exercise 3. Three dice are tossed, all outcomes equally likely. Given that all three dice have different outcomes, what is the probability that one of them will have the number 6?

(*)Exercise 4. Let A, B, C be mutually independent events in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Show that

$$\mathbb{P}(A \cup B \cup C) = 1 - \mathbb{P}(A^c) \mathbb{P}(B^c) \mathbb{P}(C^c).$$

Exercise 5. There are two fair dice, A and B . Die A has 4 red sides and 2 blue sides, and die B has 4 blue sides and 2 red sides.

A coin is tossed. If it comes out heads, we toss die A twice, and if the coin comes out tails, we toss die B twice.

- (a) The coin is a fair coin (probability $1/2$ for each side). Given that the outcome of the die is twice blue, what is the probability that we tossed die A ?
- (b) The coin is an unfair coin, with probability $3/4$ for heads and $1/4$ for tails. Given that the outcomes of the die are first blue and then red, what is the probability that we tossed die B ?
- (c) We don't know if the coin is fair or not. We want to calculate the probability it comes out heads. We know that the probability of blue-blue is $1/4$. What is the probability the coin lands heads?

Exercise 6. In an exam, there are 4 questions. The probability to answer correctly question number k is $1 - k/10$. The answers of the questions are all mutually independent events.

What is the probability that at least 2 questions are answered correctly? What is the probability that exactly 1 question is answered correctly?

Exercise 7. A fair die is tossed 5 times. All tosses are mutually independent.

What is the probability that the number 6 is observed at most once? What is the probability that the number 1 is observed at most once, conditioned on the fact that it was observed at most 4 times?