Exercise 1. Show that if $X_n \xrightarrow{\mathcal{D}} c$ where c is a constant, then $X_n \xrightarrow{P} c$.

Exercise 2. Give an example of a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and random variables on this space $(X_n)_n, X, Y$ such that

$$X_n \xrightarrow{\mathcal{D}} X \ , \qquad X_n \xrightarrow{\mathcal{D}} Y \qquad \text{and} \qquad \mathbb{P}[X = Y] = 0.$$

Exercise 3. Let $X_1, X_2, \ldots, X_n, \ldots$, be identically distributed and mutually independent random variables, such that $\mathbb{E}[X_n] = \mu$ and $\mathrm{Var}[X_n] = \sigma^2$. Let

$$S_N = \sum_{n=1}^{N} X_n.$$

Then,

$$\frac{S_N - N\mu}{\sqrt{N}\sigma} \xrightarrow{\mathcal{D}} N(0, 1).$$