Department of Mathematics, BGU

Colloquium

On *Tuesday, December ,1 2015*

At *14:30 – 15:30*

In *Math 101-*

Tomer Schlank (Hebrew University)

will talk about

Ultra-Products and Chromatic Homotopy Theory

Abstract: The category of spectra is one of the most important constructions in modern algebraic topology. It appears naturally in the study of cobordism classes of manfiolds, aa the classfiication of generalized cohomology theories and also can be thought of as a homotopical analog of abelian groups. In the last years J. Lurie and other authors began redeveloping algebra with Spectra taking the role of abelian groups. Analogs of commutative and non-commutative rings , modules, lie-algebras and many others developed, and many theorems where proved that are analogs of the classical case. I'll describe some of the tools and the ideas appearing in this constructions and sketch some applications. The same way one can do algebra in dffierent characteristics (a prime p or zero) which appear as points of the scheme $Spec(\mathbb{Z})$, One can find all possible "characteristics" of Spectra. Those are classfiied by a pair (p, n) where p is a prime and n is a natural number called a height. Working in a given characteristic (n, p) one obtains what is called the $K(n)$-local category at

height n and prime p. It is a well known observation that for a given height n certain “special” phenomena happen only for small enough primes. Further, in some sense, the categories $C_{p,n}$ become more regular and algebraic as p goes to infinity for a fixed n. The goal of this talk is to make this intuition precise.

Given an infinite sequence of mathematical structures, logicians have a method to construct a limiting one by using ultra-product. We shall define a notion of “ultraproduct of categories” and then describe a collection of categories $D_{n,p}$ which will serve as algebro-geometric analogs of the $K(n)$-local category at the prime p.

Then for a fixed height n we prove:

$$\prod_{p}^{\text{Ultra}} C_{n,p} \cong \prod_{p}^{\text{Ultra}} D_{n,p}$$

If time permits we shall describe our ongoing attempts to use these methods to get a version of the $K(n)$-local category corresponding to formal Drinfeld modules (instead of formal groups).

This is a joint project with N. Stapleton and T. Barthel.