

Department of Mathematics, BGU

Probability and ergodic theory (PET)

On Tuesday, December 22 2015

At 10:50 – 12:00

In Math 101-

Guy Cohen (BGU)

will talk about

REMARKS ON RATES OF CONVERGENCE OF POWERS OF CONTRACTIONS

Abstract: We prove that if the numerical range of a Hilbert space contraction T is in a certain closed convex set of the unit disk which touches the unit circle only at 1 then $\|T^n(I-T)\| = \mathcal{O}(1/n^\beta)$ with $\beta \in [\frac{1}{2}, 1]$. For normal contractions the condition is also necessary. Another sufficient condition for $\beta = \frac{1}{2}$, necessary for T normal, is that the numerical range of T be in a disk $\{z: |z-\delta| \leq 1-\delta\}$ for some $\delta \in (0,1)$. As a consequence of results of Sefiart, we obtain that a power-bounded T on a Hilbert space satisfies $\|T^n(I-T)\| = \mathcal{O}(1/n^\beta)$ with $\beta \in (0,1)$ if and only if $\sup_{1 < |\lambda| < 2} |\lambda - 1|^{1/\beta} |R(\lambda, T)| < \infty$. When T is a contraction on L_2 satisfying the numerical range condition, it is shown that $\|T^n\|/n^{1-\beta}$ converges to 0 a.e. with a maximal inequality, for every $f \in L_2$. An example shows that in general a positive contraction T on L_2 may have an $f \geq 0$ with $\limsup T^n f / \log n \sqrt{n} = \infty$ a.e.