Department of Mathematics, BGU

Algebraic Geometry and Number Theory

On *Wednesday, December ,7 2016*

At *15:10 – 16:30*

In *Math 101-*

Amnon Yekutieli (BGU)

will talk about

The Derived Category of Sheaves of Commutative DG Rings

Abstract:

Ben Gurion University – Mathematics
Algebraic Geometry and Number Theory Seminar

Speaker	**Amnon Yekutieli (BGU)**
Title	**The Derived Category of Sheaves of Commutative DG Rings (abstract)**
Date	Wednesday, 7 December 2016
Time	15:10 – 16:30 (starts 15:10 sharp)
Location	Room -101 in Building 58

Abstract

In modern algebraic geometry we encounter the notion of derived intersection of subschemes. This is a sophisticated way to encode what happens when two subschemes Y_1 and Y_2 of a given scheme X intersect non-transversely. The classical intersection multiplicity can be extracted from the derived intersection.

When the ambient scheme X is affine, it is not too hard to describe the derived intersection, by taking flat DG ring resolutions of the structure sheaves of the subschemes Y_1 or Y_2. This also works when the scheme X is quasi-projective. However, derived intersections in more general schemes X could only be treated using the very difficult homotopical methods of derived algebraic geometry .

Several months ago I discovered a "cheap" way to construct flat resolutions of sheaves of rings. The resolutions are by semi-pseudo-free sheaves of DG rings. The main advantage is that the geometry does not change: all the action takes place on the original topological space X.

Using semi-pseudo-free resolutions it is possible to produce derived intersections as above. It is also possible to get a direct presentation of the cotangent complex of a scheme (at least in characteristic 0). Presumably the derived adic completion of Shaul, so far studied only in the affine case, can be globalized using our our methods.

Lastly, the semi-pseudo-free resolutions give rise to a congruence on the category of sheaves of commutative DG rings on a space X, that we call relative quasi-homotopy. The functor from the homotopy category to the derived category turns out to be a faithful right Ore localization. This fact gives tight control on the derived category. It should be noted that in this situation there does not seem to exist a Quillen model structure, so the traditional approaches would fail.

In the talk I will explain the various ideas listed above. More details can be found in the eprint arxiv:1608.04265.

(updated 26 Nov 2016)