Department of Mathematics, BGU

Logic, Set Theory and Topology

On *Tuesday, December ,12 2017*

At *12:15 – 13:30*

In *Math 101-*

Denis Saveliev (Moscow, Russia)

will talk about

Hindman's finite sums theorem and its application to topologization of algebras

Abstract: In the first part of the talk I briefly outline Hindman's finite sums theorem, a famous Ramsey-theoretic result in algebra, its precursors and some generalizations, and the algebra of ultrafilters, a powerful technique providing a tool for getting similar results in combinatorics, algebra, and dynamics, see .[1] In the second (main) part I apply a multidimensional generalization of Hindman's theorem (proved by Hindman and Bergelson) to show topologizability of certain algebras.

The topologization problem for groups and rings was first posed by Markov Jr. and then studied by various authors. I consider universal algebras consisting of an Abelian group and a family of additional operation (of arbitrary arity) distributive w.r.t. the group addition. Such algebras are called here polyrings; their instances include rings, modules, algebras over a field, dffierential rings, etc. Given a polyring K, a closed subbasis of the Zariski topology on the

Cartesian product K^n consists of finite unions of sets of roots of equations $t(x_1,\ldots,x_n)=0$ for all terms t in n variables.

The main theorem (a proof of which I plan to sketch) states that, for every infinite polyring K and every $n>0$, sets definable by terms in $<n$ variables are nowhere dense in the space K^n. In particular, K^n is nowhere dense in K^{n+1}. A fortiori, all the spaces K^n are non-discrete (this fact was earlier stated by Arnautov for K a ring and $n=1$). For more details, see .[2]

References [1] N. Hindman, D. Strauss, Algebra in the Stone-Cech compactification: Theory and applications, W. de Gruyter, 2nd ed., .2012 [2] D. I. Saveliev, On Zariski topologies on polyrings, Russian Math. Surveys, 72:4 ,(2017) in press.