

Department of Mathematics, BGU

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# Operator Algebras and Operator Theory

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On Monday, December 4, 2017

At 16:00 – 17:00

In 101-

Gregory Marx (BGU)

will talk about

## Completely Positive Noncommutative Kernels, part 2

Abstract: It is well known that a function  $K : \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{Y})$  (where  $\mathcal{L}(\mathcal{Y})$  is the set of all bounded linear operators on a Hilbert space  $\mathcal{Y}$ ) being (1) a positive kernel in the sense of Aronszajn (i.e.  $\sum_{i,j=1}^N \langle K(\omega_i, \omega_j) y_j, y_i \rangle \geq 0$  for all  $\omega_1, \dots, \omega_N \in \Omega$ ,  $y_1, \dots, y_N \in \mathcal{Y}$ , and  $N = 1, 2, \dots$ ) is equivalent to (2)  $K$  being the reproducing kernel for a reproducing kernel Hilbert space  $\mathcal{H}(K)$ , and (3)  $K$  having a Kolmogorov decomposition  $K(\omega, \zeta) = H(\omega)H(\zeta)^*$  for an operator-valued function  $H : \Omega \rightarrow \mathcal{L}(\mathcal{X}, \mathcal{Y})$  where  $\mathcal{X}$  is an auxiliary Hilbert space.

Last time, I introduced free noncommutative function theory and wrote down the analogue of the result above for noncommutative kernels. In part two, I will give a sketch of our proof and discuss some well-known results (e.g. Stinespring's dilation theorem for completely positive maps) which follow as corollaries. With any remaining time, I will talk about applications and more recent related results.