

Department of Mathematics, BGU

Operator Algebras and Operator Theory

On Monday, January, 8 2018

At 16:00 – 17:00

In 101-

Jurij Volcic (BGU)

will talk about

Regular and positive noncommutative rational functions

Abstract: Hilbert's 17th problem asked whether a multivariate polynomial, which is positive on all tuples of real numbers, can be written as a sum of squares of rational functions. The positive answer was given by Artin, and the proof techniques presented a cornerstone for real algebra and real algebraic geometry. At the beginning of the millennium, Helton and McCullough solved a free version of H17: if a noncommutative polynomial is positive semidefinite on all tuples of symmetric matrices, then it can be written as a sum of hermitian squares of noncommutative polynomials.

In this talk we shall address the variation of this problem for noncommutative rational functions. By assuming that a rational function is positive semidefinite on all symmetric tuples, one quietly asserts that the function is defined on all

symmetric tuples. Such functions are called regular. We will present a characterization of regular noncommutative rational functions in terms of their realizations (from control theory) that can be algorithmically checked. Then we will discuss the proof of the rational version of Helton-McCullough theorem, and its reliance on a “truncated” GNS construction.