Department of Mathematics, BGU

AGNT

On *Wednesday, January ,22 2020*

At *15:00 − 16:15*

In *101-*

Amnon Yekutieli (BGU)

will talk about

Commutative DG Rings and their Derived Categories

Abstract: The commutative DG rings in the title are more commonly known as "nonpositive strongly commutative unital dffierential graded cochain K-algebras", where K is a commutative base ring. In the literature the standard assumption is that K is a field of characteristic zero - but one of our themes in this talk is that this assumption is superfluous (K = Z works just as well).

There are two kinds of derived categories ralated to commutative DG rings. First, given a DG ring A, we can consider D(A), the derived category of DG A-modules, which is a K-linear triangulated category. This story is well understood by now, and I will only mention it briefly.

In this talk we shall consider another kind of derived category. Let DGRng denote the category whose objects are the commutative DG rings (the base K is implicit), and whose morphisms are the DG ring homomorphisms. The derived category of commutative DG rings is the category D(DGRng) gotten by inverting

all the quasi-isomorphisms in DGRng. (In homotopy theory the convention is to call it the "homotopy category", but this is an unfortunate historical accident.)

I will define semi-free DG rings, and prove their existence and lfiting properties. Then I will introduce the quasi-homotopy relation on DGRng, giving rise to the quotient category K(DGRng), the "genuine" homotopy category. One of the main results is that the canonical functor from K(DGRng) to D(DGRng) is a faitfhul right Ore localization.

I will conclude with a theorem on the existence of the left derived tensor product inside D(DGRng), and with the pseudofunctor from D(DGRng) to the TrCat, sending a DG ring A to the triangulated category D(A).

Next semester I will talk about the geometrization of these ideas: "The Derived Category of Sheaves of Commutative DG Rings".