Department of Mathematics, BGU

Jerusalem - Be'er Sheva Algebraic Geometry Seminar

On *Wednesday, November ,18 2020*

At *15:00 – 16:30*

In

Amnon Yekutieli (Be'er Sheva)

will talk about

Rigidity, Residues and Duality: Recent Progress

Abstract: Let K be a regular noetherian ring. I will begin by explaining what is a rigid dualizing complex over an essentially finite type (EFT) K-ring A. This concept was introduced by Van den Bergh in the 1990's, in the setting of non-commutative algebra. It was imported to commutative algebra by Zhang and mysefl around ,2005 where it was made functorial, and it was also expanded to the arithmetic setting (no base field). The arithmetic setting required the use of DG ring resolutions, and in this aspect there were some major errors in our early treatment. These errors have recently been corrected, in joint work with Ornaghi and Singh. Moreover, we have established the forward functoriality of rigid dualizing complexes w.r.t essentially etale ring homomorphisms, and their backward functoriality w.r.t. finite ring homomorphisms. These results mean that we have a twisted induction pseudofunctor, constructed in a totally algebraic way (rings only, no geometry). Looking to the future, we plan to study

a more refined notion: rigid residue complexes. These are complexes of quasi-coherent sheaves in the big etale site of EFT K-rings, and they admit backward functoriality, called ind-rigid traces, w.r.t. arbitrary ring homomorphisms. Rigid residue complexes can be easily glued on EFT K-schemes, and they still have the ind-rigid traces w.r.t. arbitrary scheme maps. The twisted induction now becomes the geometric twisted inverse image pseudofunctor f \mapsto f^!. We expect to prove the Rigid Residue Theorem and the Rigid Duality Theorem for proper maps of EFT K-schemes, thus recovering almost all of the theory in the original book “Residues and Duality”, in a very explicit way. The etale functoriality implies that every finite type Deligne-Mumford (DM) K-stack admits a rigid residue complex. Here too we have the f \mapsto f^! pseusofunctor. For a map of DM stacks there is the ind-rigid trace. Under a mild technical condition, we expect to prove the Rigid Residue Theorem for proper maps of DM stacks, and the Rigid Duality Theorem for such maps that are also tame. Lecture notes will be available at http://www.math.bgu.ac.il/∼amyekut/lectures/RRD-2020/notes.pdf . (November (2020