Department of Mathematics, BGU

Colloquium

On *Tuesday, January ,10 2023*

At *14:30 – 15:30*

In *Math 101-*

Emanuel Milman (Technion)

will talk about

Multi-Bubble Isoperimetric Problems - Old and New

Abstract: The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets (“bubbles”) of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the n-sphere $\mathbb{S}^n$ and on n-dimensional Gaussian space $\mathbb{G}^n$ (i.e. $\mathbb{R}^n$ endowed with the standard Gaussian measure). Furthermore, one may consider the “multi-bubble” isoperimetric problem, in which one prescribes the volume of $p \geq 2$ bubbles (possibly disconnected) and minimizes their total surface area – as any mutual interface will only be counted once, the bubbles are now incentivized to clump together. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $p=1$; the case $p=2$ is called the double-bubble problem, and so on.

In ,2000 Hutchings, Morgan, Ritor'e and Ros resolved the double-bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently resolved in $\mathbb{R}^n$ as well) – the boundary of a minimizing double-bubble is given by three spherical caps meeting at 120°-degree angles. A more general conjecture of J.~Sullivan from the 1990's asserts that when $p \leq n+1$, the optimal multi-bubble in $\mathbb{R}^n$ (as well as in $\mathbb{S}^n$) is obtained by taking the Voronoi cells of $p+1$ equidistant points in $\mathbb{S}^{n}$ and applying appropriate stereographic projections to $\mathbb{R}^n$ (and backwards).

In ,2018 together with Joe Neeman, we resolved the analogous multi-bubble conjecture for $p \leq n$ bubbles in Gaussian space $\mathbb{G}^n$ – the unique partition which minimizes the total Gaussian surface area is given by the Voronoi cells of (appropriately translated) $p+1$ equidistant points. In the talk, we describe our approach in that work, as well as recent progress on the multi-bubble problem on $\mathbb{R}^n$ and $\mathbb{S}^n$. In particular, we show that minimizing bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$ are always spherical when $p \leq n$, and we resolve the latter conjectures when in addition $p \leq 5$ (e.g. the triple-bubble conjectures when $n\geq 3$ and the quadruple-bubble conjectures when $n\geq .(4$