

Department of Mathematics, BGU

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# Operator Algebras and Operator Theory

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*On Monday, November ,28 2022*

*At 16:00 – 17:00*

*In 101- (basement)*

Prahlad Deb (BGU)

will talk about

## **NC Gleason problem and its application in the NC Cowen-Douglas class**

Abstract: In this talk, I will discuss a noncommutative (nc) analogue of the Gleason problem and its application in the “NC Cowen-Douglas” class. The Gleason problem was first studied by Andrew Gleason in studying the maximal ideals of a commutative Banach algebra. In particular, he showed that if the maximal ideal consisting of functions in the Banach algebra  $\mathcal{A}(\mathbb{B}(0, 1))$  vanishing at the origin is finitely generated then it has to be generated by the coordinate functions where  $\mathcal{A}(\mathbb{B}(0, 1))$  is the Banach algebra of holomorphic functions on the open unit ball  $\mathbb{B}(0, 1)$  at 0 in  $\mathbb{C}^n$  which can be continuously extended up to the boundary. The question – whether the maximal ideals in algebras of holomorphic functions are generated by the coordinate functions – has been named the Gleason problem. It turns out that the existence of a local solution of the Gleason

problem in a reproducing kernel Hilbert space provides a sufficient condition for the membership of the tuple of adjoint of multiplication operators by coordinate functions in the Cowen-Douglas class.

After briefly discussing these classical aspects of the Gleason problem, I will introduce its nc counterpart for uniformly analytic nc functions and show that such a problem in the nc category is always locally uniquely solvable unlike the classical case. As an application one obtains a characterization of nc reproducing Hilbert spaces of uniformly analytic nc functions on a nc domain in  $\mathbb{C}_{\text{nc}}^d$  so that the adjoint of the  $d$  - tuple of left multiplication operators by the nc coordinate functions are in the nc Cowen-Douglas class. Along the way, I will recall necessary materials from nc function theory.

This is a part of my ongoing work jointly with Professor Vinnikov on the nc Cowen-Douglas class.