Department of Mathematics, BGU

Operator Algebras and Operator Theory

On *Monday, December ,5 2022*

At *16:00 − 17:00*

In *101- (basement)*

Prahllad Deb (BGU)

will talk about

NC Gleason problem and its application in the NC Cowen-Douglas class - ctd.

Abstract: (Part 2 of the talk from last week.)

In this talk, I will discuss a noncommutative (nc) analogue of the Gleason problem and its application in the "NC Cowen-Douglas" class. The Gleason problem was first studied by Andrew Gleason in studying the maximal ideals of a commutative Banach algebra. In particular, he showed that fi the maximal ideal consisting of functions in the Banach algebra $\mathcal{A}(\mathbb{B}(0,1))$ vanishing at the origin is finitely generated then it has to be generated by the coordinate functions where $\mathcal{A}(\mathbb{B}(0,1))$ is the Banach algebra of holomorphic functions on the open unit ball $\mathbb{B}(0,1)$ at 0 in $\mathbb{C}^n$ which can be continuously extended up to the boundary. The question – whether the maximal ideals in algebras of holomorphic functions are generated by the coordinate functions – has been named the

Gleason problem. It turns out that the existence of a local solution of the Gleason problem in a reproducing kernel Hilbert space provides a sufficient condition for the membership of the tuple of adjoint of multiplication operators by coordinate functions in the Cowen-Douglas class.

After briefly discussing these classical aspects of the Gleason problem, I will introduce its nc counterpart for unfiormly analytic nc functions and show that such a problem in the nc category is always locally uniquely solvable unlike the classical case. As an application one obtains a characterization of nc reproducing Hilbert spaces of unfiormly analytic nc functions on a nc domain in $\mathbb{C}^d_{nc}$ so that the adjoint of the d - tuple of left multiplication operators by the nc coordinate functions are in the nc Cowen-Douglas class. Along the way, I will recall necessary materials from nc function theory.

This is a part of my ongoing work jointly with Professor Vinnikov on the nc Cowen-Douglas class.