

Department of Mathematics, BGU

Colloquium

On *Tuesday, June ,28 2016*

At *14:30 – 15:30*

In *Math 101-*

Michael Lin (BGU)

will talk about

Positive Ritt contractions on \mathbb{L}_p

Abstract:

Positive Ritt contractions of L_p

Michael Lin, Ben-Gurion University, Israel

Joint work with Guy Cohen and Christophe Cuny

Let P be a Markov operator on (\mathbb{S}, m) with m σ -finite invariant. Then P extends to a contraction of each $L_p(m)$, $1 \leq p \leq \infty$, and by the Hopf-Dunford-Schwartz theorem (1956), $\frac{1}{n} \sum_{k=1}^n P^k f$ converges a.e. for $f \in L_p(m)$, $1 \leq p < \infty$.

An important question in the theory of Markov operators is that of convergence of $\{P^n f\}$, in norm or a.e., for every $f \in L_2(S, m)$.

In 1961 E.M. Stein proved that if the Markov operator P is self-adjoint in $L_2(m)$ with $-1 \notin \sigma(P)$, then $\{P^n f\}$ converges a.e. for $f \in L_p(m)$, $1 < p < \infty$; however, convergence may fail for $p = 1$, even when m is finite. An important step in Stein's proof is to show that $\sup_n n \|P^n(I - P)\|_2 < \infty$.

Combining Stein's proof with Akcoglu's pointwise ergodic theorem (1975), we obtain a similar result for self-adjoint positive contractions in L_2 .

A power-bounded T on a Banach space is called *Ritt* if $\sup_n n \|T^n(I - T)\| < \infty$.

Le Merdy and Xu (2012) studied Ritt contractions in *one* L_p space, $1 < p < \infty$ fixed. They proved that if T is a positive Ritt contraction on L_p , then there is a (p, p) -strong maximal inequality, and for every $f \in L_p(m)$ the sequence $\{T^n f\}$ converges a.e.

In this talk we discuss some properties of Ritt contractions on complex Banach and Hilbert spaces, and exhibit several examples of positive Ritt contractions on L_p with applications of the Le Merdy-Xu convergence theorem.