

Department of Mathematics, BGU

Logic, Set Theory and Topology

On *Tuesday, April ,5 2016*

At *12:30 – 13:45*

In *Math 101-*

Dor Marciano (HUJI)

will talk about

On the first undefinable ordinal - part 2

Abstract: The argument “there are only countably many definitions in the language of set theory, therefore there are uncountably many undefinable reals” is fallacious. The same applies for the argument suggesting that the first undefinable (first order, no parameters) ordinal must be countable, and the two statements are closely linked. In the paper “Pointwise definable models of Set Theory” by Joel Hamkins et al, countable models of set theory are constructed in which every element is definable. We shall demonstrate a couple of these constructions, and then move on to consider models in which there are undefinable ordinals. We’ll show that in well-founded models of $V=L$ there exists a dichotomy: Either the first undefinable ordinal is countable (according to the model), or it does not exist. We’ll construct well-founded models of ZFC (given very low-strength consistency hypotheses) in which the first undefinable ordinal is uncountable (according to the model). We’ll show that the consistency

strength of “The first undefinable ordinal is a cardinal” is roughly that of an inaccessible cardinal, and that the consistency strength of “The first undefinable ordinal is a regular cardinal” is roughly that of a Mahlo cardinal.

Note: The lecture will be given in two sessions. Basic knowledge of forcing is recommended, but not completely required - as notions and facts about the relevant forcings will be stated.