Department of Mathematics, BGU

Colloquium

On *Tuesday*, *May*, 17 2022

At 14:30 – 15:30

In Math 101-

Shakhar Smorodinsky (BGU)

will talk about

A Solution to Ringel's Circle Problem (1959)

Abstract: In 1959 Gerhard Ringel posed the following problem which remained open for over 60 years. Suppose we are given a finite family \$\C\$ of circles in the plane no three of which are pairwise tangent at the same point. Is it possible to always color the circles with five colors so that tangent circles get distinct colors.

When the circles are not allowed to overlap (i.e., the discs bounded by the circles are pairwise interiorly disjoint) then the number of colors that always suffice is four and this fact is equivalent to the Four-Color-Theorem for planar graphs.

We construct families of circles in the plane such that their tangency graphs have arbitrarily large girth and chromatic number. Moreover, no two circles are internally tangent and no two circles are concentric. This provides a strong negative answer to Ringel's 1959 open problem. The proof relies on a (multidimensional) version of Gallaiý theorem with polynomial constraints, which we derive using tools from Ramsey-Theory.

Joint work with James Davis, Chaya Keller, Linda Kleist and Bartosz Walczak