Department of Mathematics, BGU

Colloquium

On *Tuesday, May ,20 2025*

At *14:30 – 15:30*

In *Math 101-*

Eli Shamovich (BGU)

will talk about

Phenomenology of noncommutative polynomials

Abstract: Given a commuting d-tuple of matrices or operators, we immediately get a homomorphism from the polynomial ring in d variables. An extension of such a homomorphism is called a "functional calculus." On the other hand, viewing a commutative algebra as functions on its character space is a fruiftul approach that goes back at least to Gefland. However, matrices and polynomials tend not to commute. Hence, the natural object of study in this case is the free algebra $\mathbb{C}\langle z_1,\ldots, z_d \rangle$. By the first analogy, we will call the elements of the free algebra noncommutative polynomials. The second analogy tells us to treat them as functions. The natural analog of the affine space is the collection of all d-tuples of matrices of all sizes. We want to understand algebraic relations between noncommutative polynomials through their values. For example, given $f,g \in \mathbb{C}\langle z_1,\ldots, z_d \rangle$, what can we say about f and g, fi for every d-tuples of matrices $X = (X_1,\ldots,X_d)$, $f(X)$ has the same spectrum as $g(X)$. What fi $f(X)$ is always similar to $g(X)$? We will answer these questions and others.