אוניברסיטת
בן-גוריון בנגב

The Department of Mathematics

2016–17–A term

Course Name p-adic analysis

Course Number 201.2.0131

Course web page
`https://www.math.bgu.ac.il//en/teaching/fall2016/courses/p-adic-analysis`

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Office Hours `https://www.math.bgu.ac.il/en/teaching/hours`

Abstract

Requirements and grading[1]

I plan to cover Bernstein's Harvard notes on p-adic groups but I will try to emphasize applications of the theory to analysis on p-adic spaces and adelic spaces.

Course topics

The field of real numbers $\mathbb{R}$ is defined as the completion of the field of rational numbers $\mathbb{Q}$ with respect to the norm $\|\cdot\|$. However, there are other norms on $\mathbb{Q}$, each corresponding to a prime number p, and the completion of $\mathbb{Q}$ with respect to any such norm leads to the field of *p-adic numbers*, denoted by $\mathbb{Q}_p$. This is a topological complete field, and thus it makes sense to develop an analysis on it.

Many features of the p-adic analysis are very different from familiar ones in real analysis. For example, "the first year calculus dream" of many students comes true: A series converges if and only if its general term goes to 0. The overall picture of the p-adic analysis makes impression of a surprising and beautiful one, and easier than its real counterpart. Nowadays, the p-adic analysis has endless applications in geometry and number theory.

In this course we will study the field of p-adic numbers from different points of view, stressing similarities to and deviations from the real numbers. If time permits the culmination of the course will be Tate's thesis (1950), that uses p-adic

[1]Information may change during the first two weeks of the term. Please consult the webpage for updates

analysis to prove the meromorphic continuation of the zeta-function of Riemann and its functional equation.

1. Arithmetic of $\mathbb{Q}_p$: sums and products, square roots, finding roots of polynomials.

2. Algebraic number theory of $\mathbb{Q}_p$: finite extensions, algebraic closure of $\mathbb{Q}_p$, completion of the algebraic closure, local class field theory will be mentioned.

3. Topology of $\mathbb{Q}_p$: elementary topological properties, Euclidean models of $\mathbb{Z}_p$.

4. Analysis on $\mathbb{Q}_p$: convergence of sequences and series, radius of convergence, elementary functions $\ln_p$, $\exp_p$, the space of locally constant functions.

5. Harmonic analysis on $\mathbb{Q}_p$: characters of $\mathbb{Q}_p$, Haar measure, integration of locally constant functions, Fourier transform.

6. The ring of adeles as an object unifying $\mathbb{Q}_p$ for all p: topological properties, integration and Fourier transform, Poisson summation formula.

7. Tate's thesis.

Prerequisites: topology, algebraic structures