Differential and Integral Calculus for Information Systems

1) The system of the real numbers: the natural numbers, well order, the integers, the rational numbers, arithmetical operations and the field axioms, the order on the rational numbers, the Archimedian axiom, non completeness of the rationals, irrational numbers and the completeness of the reals, bounded sets, upper and lower bounds, maximum and minimum, supremum and infimum, rational and irrational powers, the basic inequalities (Bernoulli, Cauchy-Schwarz, the mean values, Holder, Minkowskii), the rational roots of a polynomial equation over the rational numbers.

2) Sequences and their limits, the arithmetic of limits, divergence and tending to infinity, inequalities between sequences and between their limits, the Sandwich Theorem, monotonic sequences, recursive sequences, Cantor’s Lemma, sub sequences, the Theorem of Bolzano-Weierstrass, the exponent, Cauchy’s criterion for the convergence of a sequence, upper limit, lower limit.

3) Functions of a single variable, arithmetic operations on functions, monotone functions, the elementary functions.

4) The limit of a function, the sequential definition and non sequential definition, the arithmetic of limits, one sided limits, Cauchy’s criterion, bounded functions, the order of magnitude of a function (big Oh and small Oh).

5) Continuous functions, classification of discontinuities, the mean value theorem and its applications, continuity and being monotone.

6) The derivative of a function, the graph of a function and the tangent line to the graph, the slope of the tangent line, the velocity, differentiability and continuity, the arithmetic of the differential operator in particular the Liebnitz rule, composition of functions, the chain rule, higher order derivatives, a theorem of Fermat, Roll’s Theorem, the mean value theorem of Lagrange, the mean value theorem of Cauchy, L’hospital rules, Taylor’s Theorem.

7) Local and
global maximum and minimum, inflection points, convex and concave functions, asymptotic lines, sketching