This course concerns the physical notion of phase transition, specifically in the model known as “percolation”.

We will review the main mathematical results regarding percolation and its related counterparts the Ising model, Potts model and Fortuin-Kasteleyn cluster model, starting from works of Ising and Pierles in the beginning of the 20th century and culminating in modern work of Smirnov (for which he was awarded a Fields Medal).

The topics in the course are, time permitting:

1. Percolation on graphs. Definitions and basic properties.
2. Harris’ inequality
3. van den Berg-Kesten inequality, Reimer’s inequality
4. Russo’s formula
5. Burton-Keane Theorem
6. Exponential decay of correlations in sub-critical regime
7. Planar percolation: Russo-Seymour-Welsh theory
9. Conformal invariance: Cardy-Smirnov formula on the triangular lattice
10. Percolation on groups

11. Critical percolation on non-amenable groups: BLPS