

The Department of Mathematics

2020–21–A term

Course Name Algebraic Geometry – Schemes 1

Course Number 201.2.0121

Course web page

https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-1/course_page.html

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Office Hours <https://www.math.bgu.ac.il/en/teaching/hours>

Abstract

See course web page for details

Please email me if you want to attend (even just for the first one or two lectures, to “taste” the course). I will send you the zoom link.

Requirements and grading¹

See course web page for details

Please **email me** if you want to attend (even just for the first one or two lectures, to “taste” the course). I will send you the zoom link.

¹Information may change during the first two weeks of the term. Please consult the webpage for updates

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פרופ' אמנון יקותיאל
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אוניברסיטת בן גוריון
באר שבע 84105

10 September 2020

Algebraic Geometry – Schemes 1

BGU, Fall 2020-21

Catalogue no. 201.2.0121

The course will be in English. It will continue in the Spring semester, as "Algebraic Geometry – Schemes 2".

Course web site (with up-to-date information):

https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-1/course_page.html

Prerequisites and Level. The course is intended for graduate students and advanced undergraduate students.

Participants of the course should have – ideally – familiarity with most of these topics: categories and functors, introduction to algebraic geometry (varieties over an algebraically closed field, or at least algebraic curves); commutative algebra; homological algebra; algebraic topology; and differentiable or complex analytic manifolds.

The level of the course will be calibrated – in terms of rate of progress and sophistication of the presentation – to the audience, under the assumption that they had already learned much of the material listed above.

Please send me an email if you are considering attending the course, indicating which of the topics above you have learned (in a formal course or privately), your academic status (degree and year), and whether you intend to register or just to listen.

Course Topics : (tentative, for both semesters)

1. **Categories and functors.** Definitions and examples. Natural transformations.
2. **Sheaves on topological spaces.** Sheaves of functions on topological spaces. Definitions and examples (sheaves of sets, abelian group, etc.). Stalks. Sheafification. Gluing (descent), cocycles and 1-st nonabelian cohomology. Operations on sheaves.
3. **Ringed spaces.** Definitions. Examples from differential and analytic geometry. Locally ringed spaces. Locally free sheaves, vector bundles, Picard group. Finiteness properties.
4. **Affine Schemes.** Definitions and basic properties. Morphisms. Examples from arithmetic.
5. **Schemes.** Definitions and basic properties. Closed and open subschemes. Noetherian and quasi-compact schemes. Coherent and quasi-coherent sheaves.

6. **Maps of schemes.** Fiber products and base change. Finite, finite type, flat, separated, proper and projective maps.
7. **Maps to projective space and blow-ups.** Definitions and examples. Computing the Picard group of the projective space \mathbf{P}^n .
8. **Calculating some invariants.** Sheaf cohomology, genus, etc.
9. **The functor of points and moduli spaces.**
10. **Algebraic differential calculus.** Smooth morphisms, differential forms, etc.
11. **Group schemes and their Lie algebras.**

Bibliography:

1. Hartshorne, *Algebraic Geometry*, Springer.
2. Eisenbud and Harris, *The Geometry of Schemes*, Springer.
3. Olsson, *Algebraic Spaces and Stacks*, AMS.
4. Kashiwara and Schapira, *Sheaves on Manifolds*, Springer.
5. de Jong (Ed.), *The Stacks Project*, [online](#)
6. [Course lecture notes](#) (to be posted weekly).



Course topics

1. Sheaves on topological spaces
2. Affine schemes
3. Schemes and morphisms between them.
4. Quasi-coherent sheaves
5. Separated and proper morphisms.
6. Vector bundles and the Picard group of a scheme.
7. The functor of points and moduli spaces.
8. Morphisms to projective space and blow-ups.
9. Smooth morphisms and differential forms.
10. Sheaf cohomology.
11. Group schemes.