

# The Department of Mathematics

2022–23–A term

**Course Name** Algebraic Geometry 1

**Course Number** 201.2.0031

**Course web page**

[https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1\\_2022-23](https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1_2022-23)

**Lecturer** Prof. Amnon Yekutieli, <amyekut@bgu.ac.il>, Office 202

**Office Hours** <https://www.math.bgu.ac.il/en/teaching/hours>

## Abstract

This is a course on algebraic varieties over an algebraically closed field. Technically it is much easier than a course on schemes, but it does not touch upon arithmetic geometry. We shall cover most of the standard material, with some additional glances into more advanced or specialized topics. The topics listed below will be adapted – to some extent – to the background and capabilities of the registered students.

See course web page<sup>1</sup> for all details

**All interested students are requested to contact the lecturer by email** before the first lecture, at amyekut@gmail.com, for the zoom link and more information.

## Requirements and grading<sup>2</sup>

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<sup>1</sup>[https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1\\_2022-23](https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1_2022-23)

<sup>2</sup>Information may change during the first two weeks of the term. Please consult the webpage for updates

# Algebraic Geometry 1

Semester A (Fall) 2022-23

*Course name:* Algebraic Geometry 1

*Catalogue no:* 201.2.0301

*Schedule:* Semester A (Fall) 2022-23

*General Description.* This is a course on algebraic varieties over an algebraically closed field. Technically it is much easier than a course on schemes, but it does not touch upon arithmetic geometry. We shall cover most of the standard material, with some additional glances into more advanced or specialized topics. The topics listed below will be adapted – to some extent – to the background and capabilities of the registered students.

*Audience:* The course is intended for graduate students at BGU. Strong undergraduate students, and outside students, can attend with special permission by the lecturer.

*Organization:* The course will span two semesters (2 hours a week each semester). The list of topics below is for both parts of the course.

*Time and place:* Wednesdays 13-15, either in room 58-201 or on Zoom.

*Grades:* The course grades are pass/fail. Passing requires attending all lectures and submitting almost all the homework assignments. Homework will be assigned each week.

*Course web page:*

[https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1\\_2022-23](https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1_2022-23)

*Language:* English.

*Prerequisite courses:*

1. "Introduction to Commutative Algebra" or "Commutative Algebra"
2. "Introduction to Topology"

*Recommended knowledge:* Categories and functors; algebraic topology; differentiable manifolds; Galois theory; homological algebra; complex analysis.

1. **Categories and functors.** Either a review (if the students are prepared), or a systematic but gradual study, mixed in with the other topics.
2. **Ringed spaces.** Topological spaces equipped with sheaves of rings of functions. Maps between ringed spaces. Study familiar examples: topological spaces and differentiable manifolds.
3. **Recalling commutative algebra.** Noetherian rings, Hilbert Basis Theorem, algebraic and transcendental field extensions, Hilbert Nullstellensatz. Localization of rings. Prime ideals. Tensor products of rings. (Students are expected to know this material.)
4. **Affine algebraic varieties.** (All varieties are over an algebraically closed field.) The Zariski topology and the sheaf of algebraic functions. Maps of affine varieties, fibered products, subvarieties, local rings. Dimension of an affine variety and its function field.
5. **Projective algebraic varieties.** Definition by homogeneous coordinate rings and by gluing. Fibered products of projective varieties. Quasi-projective varieties.
6. **Algebraic varieties.** The separation condition; varieties that are not quasi-projective.
7. **Types of maps between varieties.** Finite, quasi-finite, flat, affine and projective maps.
8. **The classification of curves.** Nonsingular projective curves and their function fields. Elliptic curves (brief).
9. **Enumerative geometry.** Bézout Theorem. Some intersection theory. A brief survey of modern methods.
10. **Vector bundles.** Definition, examples. Gluing vector bundles.
11. **Line bundles and the Picard group.** Ample line bundles and morphisms to projective space. The automorphism group of projective space.
12. **Birational geometry.** Blowups. Some examples. Singularities and their resolutions. A brief survey of modern methods.
13. **Sheaves of modules.** From locally free sheaves to vector bundles and back. Coherent sheaves. Serre's Theorem on affine open sets.
14. **Sheaf cohomology** (brief). Riemann-Roch Theorem for curves. A quick tour of Serre Duality.
15. **Differential algebraic geometry** (brief). The tangent bundle, derivations, differential forms. Étale and smooth maps.
16. **Algebraic groups** (brief). Some examples. Lie algebras. Group actions, quotients.

*Course Notes:* Typed course notes will be available every week on the course web page. These will be based partly on the books listed below, and mostly on the lecturer's preference of teaching.

1. Hartshorne, "Algebraic Geometry", Springer, 1977.
2. Gathmann, "Algebraic Geometry" (2022), [free online book](#).
3. Reid, "Undergraduate Algebraic Geometry" (2013), [free online book](#).
4. Liu, "Algebraic Geometry and Arithmetic Curves", Oxford, 2002.
5. Vakil, "THE RISING SEA – Foundations of Algebraic Geometry" (2017), [free online book](#).
6. Altman and Kleiman, "A Term of Commutative Algebra" (2021), [free online book](#).
7. D. Eisenbud, "Commutative Algebra", GTM 150, Springer, 1995.

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פרופ' אמנון יקותיאלי  
 המחלקה למתמטיקה  
 אוניברסיטת בן גוריון  
 באר שבע 84105

21/07/2022

**קורס מתקדם**  
**גיאומטריה אלגברית 1**  
**סמסטר א' (סתו) תשפ"ג**

שם הקורס: גיאומטריה אלגברית 1

מספר קטלוגי: 201.2.0301

מועד: סמסטר א' (סתו) תשפ"ג

תקציר: הקורס יעסוק ביריעות אלגבריות מעל שדה סגור אלגברית. לא נעסוק בסכמות, ולכן לא ניגע בגיאומטריה אריתמטית. הקורס יכסה את רוב החומר הסטנדרטי, עם כמה הצעות לנושאים מתקדמים יותר. תוכן הקורס יותאם - במידת מה - לרקע וליכולת של הסטודנטים הרשומים.

קהל יעד: תלמידי מחקר באב"ג. השתתפות של תלמידי תואר ראשון חזקים, ושל מאזינים מחוץ לאב"ג, מותנית באישור המרצה.

ארגון: הקורס ימשך על פני שני סמסטרים (קורס ההמשך הוא גיאומטריה אלגברית 2). רשימת הנושאים למטה היא עבור שני חלקי הקורס.

ציננים: מילוליים (עובר/נכשל). מעבר הקורס מחייב השתתפות בכל ההרצאות והגשת מרבית שיעורי הבית.

שפת הקורס: אנגלית

דרישות קדם:

1. הקורסים "מבוא לאלגברה קומוטטיבית" מס' 201.1.7071 או "אלגברה קומוטטיבית" מס' 201.2.2011.
2. הקורס "מבוא לטופולוגיה" מס' 201.1.0091.

1. קטגוריות ופונקטורים.
2. מרחבים עם חוגים.
3. חזרה על אלגברה קומוטטיבית.
4. ירועות אלגבריות אפיניות.
5. יריעות אלגבריות פרויקטיביות.
6. יריעות אלגבריות.
7. סוגים של העתקות בין יריעות.
8. מיון של עקומים.
9. גיאומטריה אינומרטטיבית.
10. אגדים ו'קטוריים.
11. אגדים קוויים ו'חבורת פיקאר.
12. גיאומטריה בירציונלית.
13. אלומות של מודולים.
14. קוהומולוגיה של אלומות.
15. גיאומטריה אלגברית דיפרציאלית.
16. חבורות אלגבריות.

## Course topics

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The course will span two semesters (2 hours a week each semester). The list of topics below is for both parts of the course. Course Topics: (For both semesters, as much as time permits. Anticipated range: until topic 10.)

1. Categories and functors. Either a review (if the students are prepared), or a systematic but gradual study, mixed in with the other topics.
2. Ringed spaces. Topological spaces equipped with sheaves of rings of functions. Maps between ringed spaces. Study familiar examples: topological spaces and differentiable manifolds.
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