אוניברסיטת
בן-גוריון בנגב

The Department of Mathematics

2016–17–B term

Course Name Noncommutative algebra

Course Number 201.2.5121

Course web page
`https://math.bgu.ac.il//en/teaching/spring2017/courses/noncommutative-algebra`

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Office Hours `https://math.bgu.ac.il/en/teaching/hours`

Abstract

Requirements and grading[1]

The course provides an introduction to the theory of non-commutative rings, and related structures. We will take as our motivating goal understanding the representation theory of finite groups. This will lead us to study the structure of *semisimple* rings, which is well understood due to a number of theorems by Wedderburn.

After drawing conclusions for representation theory, we will study in more details the building blocks in Wedderburn's theory, namely, *division rings* and rings of matrices over them. This has applications in geometry and number theory, which we will try to outline.

Further topics will vary depending on the time constraints and the taste of the audience, but may include further study of group representations, more general classes of rings (non semisimple, non Artinian) and localisation. Whenever possible, we will try to include applications and relations to other fields.

Background required: A reasonable understand of linear algebra and basic Galois theory. Familiarity with the basic notions in commutative algebra and category theory is desirable, but not required (missing material will be reviewed as needed).

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[1] Information may change during the first two weeks of the term. Please consult the webpage for updates

Course topics

.1 Basic Algebraic Structures: rings, modules, algebras, the center, idempotents, group rings

.2 Division Rings: the Hamiltonian quaternions, generalized quaternion algebras, division algebras over $\mathbb{F}_q$, $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Q}$ (theorems of Frobenius and Wedderburn), cyclic algebras, the Brauer–Cartan–Hua theorem

.3 Simplicity and semi-simplicity: simplicity of algebraic structures, semi-simple modules, semi-simple rings, Maschke's theorem

.4 The Wedderburn–Artin Theory: homomorphisms and direct sums, Schur's lemma, the Wedderburn–Artin structure theorem, Artinian rings

.5 Introduction to Group Representations: representations and characters, applications of the Wedderburn–Artin theory, orthogonality relations, dimensions of irreducible representations, Burnside's theorem

.6 Tensor Products: tensor products of modules and algebras, scalar extensions, the Schur index, simplicity and center of tensor products, the Brauer group, the Skolem–Noether theorem, the double centralizer theorem, maximal fields in algebras, reduced norm and trace, crossed products