Field Theory and Galois Theory

Dr Ishai Dan-Cohen

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• Fields: basic properties and examples, the characteristic, prime fields
• Polynomials: irreducibility, the Eisenstein criterion, Gauss’s lemma
• Extensions of fields: the tower property, algebraic and transcendental extensions, adjoining an element to a field
• Ruler and compass constructions
• Algebraic closures: existence and uniqueness
• Splitting fields
• Galois extensions: automorphisms, normality, separability, fixed fields, Galois groups, the fundamental theorem of Galois theory.
• Cyclic extensions
• Solving polynomial equations by radicals: the Galois group of a polynomial, the discriminant, the Cardano-Tartaglia method, solvable groups, Galois theorem
• Roots of unity: cyclotomic fields, the cyclotomic polynomials and their irreducibility
• Finite fields: existence and uniqueness, Galois groups over finite fields, primitive elements