Fourier Analysis

Dr Izhar Oppenheim

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• Cesaro means: Convolutions, positive summability kernels and Fejer’s theorem.

• Applications of Fejer’s theorem: the Weierstrass approximation theorem for polynomials, Weyl’s equidistribution theorem, construction of a nowhere differentiable function (time permitting).

• Pointwise and uniform convergence and divergence of partial sums: the Dirichlet kernel and its properties, construction of a continuous function with divergent Fourier series, the Dini test.

• $L^2$ approximations. Parseval’s formula. Absolute convergence of Fourier series of $C^1$ functions. Time permitting, the isoperimetric problem or other applications.

• Applications to partial differential equations. The heat and wave equation on the circle and on the interval. The Poisson kernel and the Laplace equation on the disk.

• Fourier series of linear functionals on $C^m(\mathbb{T})$. The notion of a distribution on the circle.

• Time permitting: positive definite sequences and Herglotz’s theorem.

• The Fourier transform: convolutions, the inversion formula, Plancherel’s theorem, Hermite functions. Time permitting: tempered distributions, further applications to differential equations.
• Fourier analysis on finite cyclic groups, and the Fast Fourier Transform algorithm.