Field Theory and Galois Theory

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• Fields: basic properties and examples, the characteristic, prime fields

• Polynomials: irreducibility, the Eisenstein criterion, Gauss’s lemma

• Extensions of fields: the tower property, algebraic and transcendental extensions, adjoining an element to a field

• Ruler and compass constructions

• Algebraic closures: existence and uniqueness

• Splitting fields

• Galois extensions: automorphisms, normality, separability, fixed fields, Galois groups, the fundamental theorem of Galois theory.

• Cyclic extensions

• Solving polynomial equations by radicals: the Galois group of a polynomial, the discriminant, the Cardano-Tartaglia method, solvable groups, Galois theorem

• Roots of unity: cyclotomic fields, the cyclotomic polynomials and their irreducibility

• Finite fields: existence and uniqueness, Galois groups over finite fields, primitive elements